## Application Conference Matrices for Parameter Design

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#### Abstract

When optimizing using an orthogonal array, it is desirable to consider the various relationships between factors and assign many factors. Two-level orthogonal array can be assigned many factors. Three-level orthogonal array has the advantage of obtaining intermediate information on the level. For this reason, mixed type orthogonal arrays $L_{18}\left(2^{1} 3^{7}\right), L_{36}\left(2^{11} 3^{13}\right)[1,2]$, etc are still used today. The response of these mixed typed orthogonal arrays is logarithmically converted to the SN ratio and sensitivity for optimization. This way also is called Taguchi methods ${ }^{[1]}$.

Parameter design with a two-step procedure for predicting the optimum conditions is performed from this SN ratio and sensitivity with factor effect graph. However, this method has two problems (1) and (2). (1): The number of experiments will be increased proportional to the number of layout factors in the mixed type orthogonal array. (2): In the first step of reducing the variation, select the combination of the levels that maximum levels the SN ratio of the factor effect graph as the optimum condition.

The confirmation value (b) had been expected as the optimum condition with minimized the variation. But, there are the problem that this confirmation value (b) is worse than the best value (a) of the SN ratio of the orthogonal array used for estimation" will be appeared for $62 \%$ of cases [3,4].

So, the prediction accuracy for the optimum conditions are poor. In order to improve these problems (1) and (2), this paper report will propose a new method to apply the conference matrix to the layout and the coefficient figure to the analysis to the row data.

This report provides an easy-to-understand explanation that the conference matrix [5-11] reduces the number of experiments and improves prediction accuracy using the Coefficient of variation, especially for researchers.

We are sure our proposed ways to reduce the experimental number and the period and cost almost to $1 / 3 \sim 1 / 2$ with the higher accuracy for optimizing, so we will recommend as the specific ways to solve the subjects of the Sustainable Development Goals. Especially it will contribute to create the effective countermeasures to Global Warning that has been requested immediately to take the actions to reduce the increasing temperature.


Keywords: Optimizing; Mixed Type Orthogonal Array; Conference Matrices; Coefficient Of Variation; Coefficient Graph; Regression Analysis; Taguchi Methods; Global Warning; Sustainable Development Goals

## 1 Introduction

The conference matrix C was used for the first time in the telephone network diagnosis of Belevitch (1950)[5]. The conference matrix (C-Matrices) was used in the 2010s by Xiao, Lin, Bai (2012) ${ }^{[7]}$. It was related to the DSD design in which the central condition C 0 was added to the overlap ( $\mathrm{C}+\mathrm{C}$-). This was noted in the optimization study because the main effect of the first-order coefficient does not involve the interaction between the two factors.

Suzuki, Tanaka, and Miyagawa [8] were used to estimate the experimental values that were not performed. And Mori, Sadamatsu, Matsuura, and Tanaka [ 10 ] combined with noise factors to reduce variability.
The requirements for the C matrix are the following three points[11].

- The diagonal component is zero.
- Each element is either -1 or +1 .
- Each column is orthogonal.

A typical C matrix has one column as two levels and the other column as three levels [7]. In the C matrix, the linear terms of the three levels are orthogonal, the number of rows is smaller than the mixed type orthogonal array with respect to the number of allocation factors. Compared between the conference matrices and the mixed orthogonal arrays, the number of experiments is $1 / 3$ or less in the case study of this report.

For example, for the allocation of 3 levels and 5 factors, $\mathrm{L}_{18}$ $\left(2^{1} 3^{7}\right)$ is used for the mixed type orthogonal array, and $C_{6}\left(2^{1} 3^{5}\right)$ is used for the C matrix. Since the number of experiments matches the number of rows in the matrix, it is $1 / 3$ from the subscript on the lower right side of $\mathrm{C}_{6}$ and $\mathrm{L}_{18}$.

In addition, if limited to a 3-level orthogonal array, the column spacing is $\mathrm{L}_{9}\left(3^{4}\right), \mathrm{L}_{18}\left(3^{7}\right), \mathrm{L}_{27}\left(3^{13}\right), \mathrm{L}_{36}\left(3^{13}\right)$, and 9 spacing. On the other hand, the C matrix exists at two intervals in principle. For this reason, it is possible to deal with various allocation factors in detail, and the number of empty columns can be reduced to zero or one.

Mori, Sadamatsu, Tomishima, Tanabe (2020) [12] also reported that the confounding of the mixed type of orthogonal array $\mathrm{L}_{18}$ affects the optimization.

It will be better for the researcher to apply to the C matrix and coefficient of variation than the mixed type of orthogonal array and SN ratio (db). Because former is smaller than latter at the effect of confounding.

2 Dynamic characteristic analysis of temperature control circuit

As a verification study, we try to compare "SN ratio (db) logarithmically transformed with mixed type orthogonal array" and "C matrix and coefficient of variation" in the dynamic characteristic case study of parameter design.

In this report, we will take the dynamic characteristic experiment of the temperature control circuit of Madhav S. Phadke [13] (1989) in the United States. Dr Phadke's book "Quality Engineering using Robust Design" (Prentice Hall, USA) is the typical study book for the parameter design in the United States.

### 2.1 Temperature control circuit and layout factor

The temperature control circuit to be verified is shown in figure 1. Response (y) is the signal voltage RT-ON of R3 when ON. The relational expression is shown in Equation [1].

The research is to reduce the variation by changing the combination level of factors which is called the parameter design.

Target value is the same response of the initial condition which was consisting with the second level which is shown as No2 of $\mathrm{L}_{18}$ or C 0 of $\mathrm{C}_{6}$.

$$
\begin{equation*}
\mathrm{R}_{\mathrm{T}-\mathrm{ON}}=\mathrm{R}_{3} \frac{\mathrm{R}_{2}\left(\mathrm{E}_{2} \mathrm{R}_{4}+\mathrm{E}_{0} \mathrm{R}_{1}\right)}{\mathrm{R}_{1}\left(\mathrm{E}_{2} \mathrm{R}_{2}+\mathrm{E}_{2} \mathrm{R}_{4}-\mathrm{E}_{0} \mathrm{R}_{2}\right)} \tag{1}
\end{equation*}
$$



Figure 1: Temperature Control Circuit (RT-ON).

### 2.2 Control factor, noise factor and signal factor

Based on the 2nd level, R1, R2, R3 were multiplied by 1.5 to the 3rd level, and then divided by 1.5 to make the 1st level. For EZ, center 6 was set as the second level, -1.2 was set to $4.8,+1.2$ was set to 7.2 , and the first and third levels were set.

E0 was fixed at 10V. There were 4 control factors ABCD, and each level was as shown in table 1. $\mathrm{E} 0(\mathrm{x})$ is not a control factor, but it is also shown in table 1.

| $\mathbf{L}_{18}$ | C $_{6}$ | R1 | R2 | R4 | Ez | E0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Level |  | A | B | C | D | X |
| 1 | -1 | 2.667 | 5.333 | 26.667 | 4.8 | 10 |
| 2 | 0 | 4 | 8 | 40 | 6 | 10 |
| 3 | 1 | 6 | 12 | 60 | 7.2 | 10 |
| Unit |  | $\mathrm{K} \Omega$ | $\mathrm{K} \Omega$ | $\mathrm{K} \Omega$ | V | V |

Table 1: Control factor ABCD.

The range of component variation was $\pm 0.024$, to 1 of the central value ABCD . We will create the N 1 and N 2 of the noise factor. The control factor is fixed at the second level In Equation [1], and the combination that changes for each factor and the response becomes smaller is N 1 (A3B1C3D3X1), and the combination that becomes larger is N2 (A1B3C1D1X3). The above is shown in table 2.

| Compound <br> noise factor | R1 | R2 | R4 | Ez | E0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | $\mathbf{X}$ |
| N1 | 1.0204 | 0.9796 | 1.0204 | 1.0204 | 0.9796 |
| N0 | 1 | 1 | 1 | 1 | 1 |
| N2 | 0.9796 | 1.0204 | 0.9796 | 0.9796 | 1.0204 |

Table 2: Noise factor.
$R 3$ is selected as the signal factor (M) judging from equation [1]. And the $0.5,1.0,1.5(\mathrm{k} \Omega)$ are set as signal level which is shown in table 3. The relationship between the signal factor (M) and the response(y) is equation [2] with $\beta$ which is a constant of proportionality.

$$
\begin{equation*}
\mathrm{y}=\mathrm{RT}-\mathrm{on}=\beta \times R 3+\mathrm{e} \tag{2}
\end{equation*}
$$

| Signal factor: R3(M) [Unit: K $\boldsymbol{]}$ ] |  |  |  |
| :---: | :---: | :---: | :---: |
| M | M1 | M2 | M3 |
| Resistance | 0.5 | 1 | 1.5 |

Table 3: Signal factors.
$2.3 \mathrm{~L}_{18}$ and $\mathrm{C}_{6}$ allocation table used
The four-factor ABCD corresponds to columns 3.4.5.7 of $\mathrm{L}_{18}$ and columns $2,3,4,5$ of the $C_{6}$ matrices. This is shown in table 4 (left, right).

### 2.4 Calculation example (applying initial conditions)

Here, the calculation process of No. 2 of $\mathrm{L}_{18}$ which is the initial condition in table 4 is shown. This also corresponds to C 0 of $\mathrm{C}_{6}$. When the second level of table 1 is transcribed to table 5. And multiplied by table 2 , it becomes table 6 left.

| $\mathbf{C o l}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{L}_{18}$ | $\mathbf{e}$ | $\mathbf{e}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{e}$ | $\mathbf{D}$ | $\mathbf{e}$ |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 1 | 2 | 1 | 1 | 2 | 2 | 3 | 3 |
| 5 | 1 | 2 | 2 | 2 | 3 | 3 | 1 | 1 |
| 6 | 1 | 2 | 3 | 3 | 1 | 1 | 2 | 2 |
| 7 | 1 | 3 | 1 | 2 | 1 | 3 | 2 | 3 |
| 8 | 1 | 3 | 2 | 3 | 2 | 1 | 3 | 1 |
| 9 | 1 | 3 | 3 | 1 | 3 | 2 | 1 | 2 |
| 10 | 2 | 1 | 1 | 3 | 3 | 2 | 2 | 1 |
| 11 | 2 | 1 | 2 | 1 | 1 | 3 | 3 | 2 |
| 12 | 2 | 1 | 3 | 2 | 2 | 1 | 1 | 3 |
| 13 | 2 | 2 | 1 | 2 | 3 | 1 | 3 | 2 |
| 14 | 2 | 2 | 2 | 3 | 1 | 2 | 1 | 3 |
| 15 | 2 | 2 | 3 | 1 | 2 | 3 | 2 | 1 |
| 16 | 2 | 3 | 1 | 3 | 2 | 3 | 1 | 2 |
| 17 | 2 | 3 | 2 | 1 | 3 | 1 | 2 | 3 |
| 18 | 2 | 3 | 3 | 2 | 1 | 2 | 3 | 1 |


| Col | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{C}_{6}$ | $\mathbf{1}$ | A | B | C | D | $\mathbf{6}$ |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 1 | 1 | -1 | -1 |
| 3 | 1 | 1 | 0 | -1 | -1 | 1 |
| 4 | 1 | 1 | -1 | 0 | 1 | -1 |
| 5 | 1 | -1 | -1 | 1 | 0 | 1 |
| 6 | 1 | -1 | 1 | -1 | 1 | 0 |

Table 4: Compared $\mathrm{L}_{18}$ (left) and $\mathrm{C}_{6}$ (right).

Substituting this value into the upper (numerator) and lower (denominator) of Equation [1] yields table 6 on the right. Multiply this upper and lower value by the signal R3 to get the left side of
table 6 . Next, convert $\Omega$ to $\mathrm{k} \Omega$ to match the signal unit. After Input it the center of table 7, then the proportionality constant $\beta$ divided by R3 are put in the right of table 7 .

| Col | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{7}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Factor | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{x}$ |
| N1 | 4000 | 8000 | 40000 | 6 | 10 |
| N0 | 4000 | 8000 | 40000 | 6 | 10 |
| N2 | 4000 | 8000 | 40000 | 6 | 10 |

Table 5: Initial (second level) combination.

| Factor | R1 | R2 | R4 | Ez | E0 | Calculation [1] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{X}$ | 上 | 下 |
| N1 | 4081.6 | 7836.8 | 40816 | 6.122 | 9.796 | 2271694218.1 | 902452412.6 |
| N0 | 4000 | 8000 | 40000 | 6 | 10 | 2240000000.0 | 832000000.0 |
| N2 | 3918.4 | 8163.2 | 39184 | 5.878 | 10.2 | 2206441385.1 | 764051205.9 |

Table 6: Effect of noise factor.

| Compound noise Factor | Signal Factor ( $\Omega$ ) |  |  | Signal Factor (K) |  |  | Signal Facto (K) : $\boldsymbol{\beta}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | M1 | M2 | M3 | M1 | M2 | M3 | M1 | M2 | M3 |
|  | 0.5 | 1 | 1.5 | 0.5 | 1 | 1.5 | 0.5 | 1 | 1.5 |
| N1 | 1258.623 | 2517.245 | 3775.868 | 1.259 | 2.517 | 3.776 | 2.517 | 2.517 | 2.517 |
| N0 | 1346.154 | 2692.308 | 4038.462 | 1.346 | 2.692 | 4.038 | 2.692 | 2.692 | 2.692 |
| N2 | 1443.909 | 2887.819 | 4331.728 | 1.444 | 2.888 | 4.332 | 2.888 | 2.888 | 2.888 |

Table 7: Responses to signal factors.

When the noise is mixed, it is analyzed by N 1 and N 2 , but Phadke obtains the SN ratio and sensitivity from the nine in the center of table 7 including N 0 .
$\mathrm{S}_{\mathrm{T}}=1.259^{2}+\ldots . .+2.888^{2}+4.332^{2}=76.7359$
$\mathrm{L}=0.5 \times 1.259+\ldots+1.5 \times 4.332=76.49534$
$r=3\left(0.5^{2}+1.0^{2}+1.5^{2}\right)=10.5$
$S_{\beta}=L^{2} /(3 r)=76.49534$
$\mathrm{Se}=\mathrm{S}_{\mathrm{T}}-\mathrm{S}_{\beta}=0.2406: \mathrm{Ve}=\mathrm{Se} / 8=0.03007$
$\beta^{2}=S_{\beta} / 3 r=7.28527$
SN Ratio(db) $=10 \times$ LOG ( $\left.\beta^{2} / \mathrm{Ve}\right)=23.8431$
Sensitivity $(\mathrm{db})=10 \times \operatorname{LOG}\left(\beta^{2}\right)=8.624457$
The analysis index of the new proposal is as follows.
Average of $\beta=(2.517+\ldots 2.888) / 9=2.6991$

Standard deviation $=[(2.517$-average $) 2+\ldots] / 8=0.160544$
Coefficient of variation $=$ standard deviation/average $=0.05948$

### 2.5 Experimental results and graphing

The experimental results of $\mathrm{L}_{18}$ are shown in table 8 and the factor effect diagram is shown in figure 2.

The experimental results by the proposed method are shown in table 9-1, and the results of the regression analysis are shown in table 9-2. The coefficients corresponding to the levels are obtained from table 9-2 and are shown in table 9-3.CV in the table averages the coefficient of variation.

| No | SN ratio | $\boldsymbol{\beta}^{\mathbf{2}}$ | Sensitivity |
| :---: | :---: | :---: | :---: |
| 1 | 22.413 | 9.588 | 9.817 |
| 2 | 23.843 | 7.285 | 8.624 |
| 3 | 24.794 | 6.124 | 7.870 |
| 4 | 25.854 | 5.327 | 7.265 |
| 5 | 24.192 | 7.120 | 8.525 |
| 6 | 19.469 | 15.673 | 11.952 |
| 7 | 22.249 | 19.292 | 12.854 |
| 8 | 23.614 | 15.044 | 11.774 |
| 9 | 24.934 | 1.418 | 1.515 |
| 10 | 24.233 | 31.249 | 14.948 |
| 11 | 24.508 | 3.065 | 4.864 |
| 12 | 22.130 | 5.029 | 7.015 |
| 13 | 26.029 | 11.316 | 10.537 |
| 14 | 16.187 | 61.093 | 17.860 |
| 15 | 24.602 | 1.493 | 1.741 |
| 16 | 20.270 | 58.409 | 17.665 |
| 17 | 25.949 | 2.487 | 3.956 |
| 18 | 23.049 | 3.949 | 5.965 |
| BM | 23.843 | 7.285 | 8.624 |

Table 8: Experimental results.


Figure 2: Initial: SN ratio (db) and sensitivity (vertical axis db).

Table 9-3 shows how to obtain the A level value of CV. The intercept ( 0.066 ) in table $9-2$ is used as the second level. The first level is 0.063 , which is the intercept $(0.066)$ minus the coefficient of A
(0.003). The third level is 0.0069 when the coefficient of A (0.003) is added to this intercept $(0.066)$. Others are calculated in the same way.

| No | Average | CV |
| :---: | :---: | :---: |
| 1 | 2.475 | 0.053 |
| 2 | 4.377 | 0.068 |
| 3 | 2.922 | 0.097 |
| 4 | 1.135 | 0.050 |
| 5 | 2.286 | 0.046 |
| 6 | 6.240 | 0.074 |
| C0 | 2.699 | 0.059 |

Table 9.1: Average and coefficient of variation.

| Item | Level | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SN Ratio <br> db | 1 | 23.508 | 24.710 | 21.313 | 21.688 |
|  | 2 | 23.049 | 23.582 | 23.386 | 23.391 |
|  | 3 | 23.163 | 21.428 | 25.022 | 24.641 |
| Sensitivity <br> db | 1 | 12.181 | 4.860 | 10.552 | 10.400 |
|  | 2 | 9.267 | 8.920 | 9.014 | 9.012 |
|  | 3 | 6.010 | 13.678 | 7.892 | 8.046 |

Table 9.2: Regression analysis for Average and coefficient of variation.

| Item | Level | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CV | 1 | 0.063 | 0.059 | 0.080 | 0.077 |
|  | 2 | 0.066 | 0.066 | 0.066 | 0.066 |
|  | 3 | 0.069 | 0.073 | 0.052 | 0.055 |
|  | 1 | 4.288 | 1.955 | 3.894 | 3.379 |
|  | 2 | 3.241 | 3.241 | 3.241 | 3.241 |
|  | 3 | 2.194 | 4.527 | 2.588 | 3.103 |

Table 9.3: Coefficients corresponding to the level.

### 2.6 Comparison of optimum conditions

Here, compare the optimum conditions of the conventional method and the proposed method.

In the conventional method, the minimum variation from the SN ratio (db) in figure 2 (top) is the maximum level combination A1B1C3D3 marked with a circle $\bigcirc$. In the proposed method, the minimum variation is from the coefficient of variation in figure 3


Figure 3: Coefficient graph (top: coefficient of variation: bottom: average).
(top). It is the lowest level combination A1B1C3D3 marked with O. The conventional method and the proposed method resulted the same conditions under the parameter design, but the number of experiments (trials in table 10) is from 18 to 6 , and it is $1 / 3$.

### 2.7 The Optimum conditions and Tune

Table 10 shows a comparison between the initial, optimum, and tune.

The average value $\beta$ of the optimum is 2.2019 , which is smaller than the initial 2.6991 . So, tuned to 2.6991 of the initial with shifting A from 2.667 to 2.151. The results are shown in figure 4.

## Summary

In this report, the response of the mixed type orthogonal array $\mathrm{L}_{18}$ is taken up as the conventional method of parameter design by dynamic characteristics: SN ratio (db) and sensitivity.

| Condition |  | Signal Factor |  |  | Disversion |  | Average |  | Trial numbers |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.5 | 1 | 1.5 | SN Ratio | CV | Sensitivity | Raw | L18 | C6 |
| Initial | N1 | 1.259 | 2.517 | 3.776 | 23.843 | 0.05948 | 8.6245 | 2.699 | 18 | 6 |
|  | N0 | 1.346 | 2.692 | 4.038 |  |  |  |  |  |  |
|  | N2 | 1.444 | 2.888 | 4.332 |  |  |  |  |  |  |
| Optimum | N1 | 1.047 | 2.093 | 3.140 | 26.6238 | 0.04319 | 6.8561 | 2.202 |  |  |
|  | N0 | 1.100 | 2.199 | 3.299 |  |  |  |  |  |  |
|  | N2 | 1.156 | 2.313 | 3.469 |  |  |  |  |  |  |
| Tune | N1 | 1.284 | 2.567 | 3.851 | 26.700 | 0.04281 | 8.6242 | 2.699 |  |  |
|  | N0 | 1.348 | 2.696 | 4.044 |  |  |  |  |  |  |
|  | N2 | 1.417 | 2.834 | 4.251 |  |  |  |  |  |  |

Table 10: Comparison of initial and optimum and adjustment.

In response to this, we proposed a parameter design based on the coefficient of variation and the average for the response of the conference matrix $\mathrm{C}_{6}$. This conventional method and the proposed method were compared and verified. Table 11 shows the results.

Results are the variation reduction was the same, but the number of experiments was 18 for the conventional method, and 6 for the proposed method, which was $1 / 3$.

## 3 Discussion

3.1 Why the conference matrix can reduce the number of experiments
$\mathrm{L}_{18}$ is a typical mixed type orthogonal array of 3 levels used in parameter design. This $\mathrm{L}_{18}$ is derived from the Bose, Bush matrix 6 rows 6 columns $\left(B_{6}\right)[14]$. In this 6 rows and 6 columns $\left(B_{6}\right)$ itself, the sum of products is not zero and the columns are not orthogonal to each other. Therefore, if -1 is expanded to " $-1,0,+1$ ", 0 to " $0,+$ $1,-1$,", and 1 to " $+1,-1,0$,", it becomes $L_{18}$., The sum of products becomes zero. In $\mathrm{C}_{6}$, the sum of products is zero even if it is not expanded, and the columns are orthogonal to each other.


Figure 4: Signal factor (Horizontal) and response (y: Vertical). Top: Initial, Middle: Optimum, Bottom: Tune.

Judging from this sum of products, in principle, $\mathrm{C}_{6}$ can obtain information on the primary main effect term of 3 levels with $1 / 3$ of the number of experiments of $L_{18}$.

Table 12 shows the conference matrices $\mathrm{C}_{6}$ and $\mathrm{B}_{6}$, and table 13 shows the sum of products of $L_{18}$.

### 3.2 The problem of logarithm conversion

Assuming that the product model $y=A^{\alpha} B^{\beta} C^{\gamma} D^{\theta}$, the logarithms conversion are taken for the better prediction both sides.
$Y=\log (y)=\alpha \log A+\beta \log B+\gamma \log C+\theta \log D$

This improves the prediction accuracy for the product model. However, the actual research subject of science is the mixed model

| B6 | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 2 | -1 | -1 | 0 | 1 | 0 | 1 |
| 3 | -1 | 0 | -1 | 1 | 1 | 0 |
| 4 | -1 | 1 | 1 | -1 | 0 | 0 |
| 5 | -1 | 0 | 1 | 0 | -1 | 1 |
| 6 | -1 | 1 | 0 | 0 | 1 | -1 |


| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
|  | 2 | -1 | 2 | -1 |
|  |  |  |  |  |
|  |  | -1 | -1 | 2 |
|  |  | 2 | 2 |  |
|  |  |  |  |  |
|  |  |  |  |  |


| C6 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 0 | 1 | 1 | -1 | -1 |
| 3 | 1 | 1 | 0 | -1 | -1 | 1 |
| 4 | 1 | 1 | -1 | 0 | 1 | -1 |
| 5 | 1 | -1 | -1 | 1 | 0 | 1 |
| 6 | 1 | -1 | 1 | -1 | 1 | 0 |


| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 |
|  |  | 0 | 0 | 0 |
|  |  |  | 0 | 0 |
|  |  |  |  | 0 |

Table 12: Sum of the product of the conference matrices $B_{6}$ and $C_{6}$.
in which an additive model and a product model are mixed. A typical example (mechanics) [15] is shown below.

$$
y=\frac{I^{2} \cdot(E+I)}{3 A\left(C^{4} / 12-\frac{\pi \cdot D^{2}}{64}\right)} \sqrt{\left.(E+F)^{2}+(G+H)^{2}\right)}
$$

Forcible logarithmic conversion causes a non-linear effect in the additive model term, which reduces the prediction accuracy. So, the coefficient of variation is applied to the proposed method by adjusting the average and using the standard deviation with a less nonlinear effect as the variation index.

| B6 | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{L 1 8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
|  | 1 | 1 | 1 | 1 | 1 | 1 | 3 |
| 2 | -1 | -1 | 0 | 1 | 0 | 1 | 4 |
|  | 0 | 0 | 1 | -1 | 1 | -1 | 5 |
|  | 1 | 1 | -1 | 0 | -1 | 0 | 6 |
| 3 | -1 | 0 | -1 | 1 | 1 | 0 | 7 |
|  | 0 | 1 | 0 | -1 | -1 | 1 | 8 |
|  | 1 | -1 | 1 | 0 | 0 | -1 | 9 |
| 4 | -1 | 1 | 1 | -1 | 0 | 0 | 10 |
|  | 0 | -1 | -1 | 0 | 1 | 1 | 11 |
|  | 1 | 0 | 0 | 1 | -1 | -1 | 12 |
|  | -1 | 0 | 1 | 0 | -1 | 1 | 13 |
|  | 0 | 1 | -1 | 1 | 0 | -1 | 14 |
| 6 | 1 | -1 | 0 | -1 | 1 | 0 | 15 |
|  | -1 | 1 | 0 | 0 | 1 | -1 | 16 |
|  | 0 | -1 | 1 | 1 | -1 | 0 | 17 |
|  | 1 | 0 | -1 | -1 | 0 | 1 | 18 |


| 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
|  | 0 | 0 | 0 | 0 |
|  |  | 0 | 0 | 0 |
|  |  | 0 | 0 |  |
|  |  |  |  |  |
|  |  |  |  |  |

Table 13: the sum of products of $L_{18}$ after expanded.

### 3.3 Evaluation for the variation without data conversion.

For the calculation of SN ratio and sensitivity, there is three-step data conversion: 1 : raw data is squared, 2 : variable decomposition is performed, and 3:further logarithmic conversion is performed.

For the coefficient of variation with less non-linear effect, data conversion of raw data is squared, then the standard deviation is calculated.

Here, we propose a "variation evaluation method" that uses a graph that does not convert data.

Table 14: $\beta \mathrm{N} 1, \beta \mathrm{~N} 0, \beta \mathrm{~N} 2$ for $\mathrm{C}_{6}$ are shown in table 14, the coefficient by level is shown in table 15, and the coefficient graph is shown in figure 5.

| C6 | A | B | C | $\mathbf{D}$ | $\boldsymbol{\beta N 1}$ | $\boldsymbol{\beta N O}$ | $\boldsymbol{\beta N 2}$ | N2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 | 1 | 2.475 | 2.325 | 2.470 | 2.629 |
| 2 | 0 | 1 | 1 | -1 | 4.377 | 4.040 | 4.362 | 4.730 |
| 3 | 1 | 0 | -1 | -1 | 2.922 | 2.607 | 2.901 | 3.258 |
| 4 | 1 | -1 | 0 | 1 | 1.135 | 1.070 | 1.133 | 1.201 |
| 5 | -1 | -1 | 1 | 0 | 2.286 | 2.168 | 2.283 | 2.408 |
| 6 | -1 | 1 | -1 | 1 | 6.240 | 5.720 | 6.212 | 6.787 |
| C0 | 0 | 0 | 0 | 0 | 2.699 | 2.517 | 2.692 | 2.888 |

Table 14: $\beta \mathrm{N} 1, \beta \mathrm{~N} 0, \beta \mathrm{~N} 2$ for $\mathrm{C}_{6}$.

| Level | $\boldsymbol{\beta} \mathbf{N} \mathbf{1}$ | $\boldsymbol{\beta} \mathbf{N} \mathbf{0}$ | $\boldsymbol{\beta} \mathbf{N} \mathbf{2}$ |
| :---: | :---: | :---: | :---: |
| A1 | 3.961 | 4.272 | 4.630 |
| A2 | 2.987 | 3.228 | 3.507 |
| A3 | 2.012 | 2.184 | 2.385 |
| B1 | 1.815 | 1.949 | 2.101 |
| B2 | 2.987 | 3.228 | 3.507 |
| B3 | 4.159 | 4.508 | 4.913 |
| C1 | 3.543 | 3.874 | 4.264 |
| C2 | 2.987 | 3.228 | 3.507 |
| C3 | 2.431 | 2.583 | 2.750 |
| D1 | 3.091 | 3.364 | 3.683 |
| D2 | 2.987 | 3.228 | 3.507 |
| D3 | 2.883 | 3.093 | 3.332 |

Table 15: The coefficient by level.


Figure 5: The coefficient graph.

The optimum conditions (circles) are obtained by combining the levels at which $\beta \mathrm{N} 1, \beta \mathrm{~N} 0$, and $\beta \mathrm{N} 2$ approach each other. The researcher at optimizing will select the levels for next step with figure 5.

Optimal conditions can be known from ordinary statistical processing without data conversion. Without data conversion, the prediction accuracy of the optimum conditions can be maintained high.

### 3.4 Layout of classification factors

Although the continuous factor of the primary term can be assigned to the conference matrix, it was considered difficult to assign the classification factor. However, if the classification factor has two levels, it may be assigned to the column consisting of [0.1] in the conference matrix.

If there are three levels, the order of level arrangement at the conference matrices will follow the order of physical chemical properties to maintain the primary term.

For example, the solvent types, material differences, catalyst types, metal components, alloy types and the like are typical classification factors.

However, when adopting these different species (classification factors) in experiments, the researchers are comparing them with numerical values of the same physicochemical properties. For example, suppose that three types of solvents, Methanol (M), Ethanol (E), and Butanol (B), are taken up. Since these are three levels of classification factors, $L_{18}$ and $L_{27}$ are generally considered for layout.

However, the boiling point (BP) of physical chemical properties is $(65,78,117)$ degrees $C$. If this boiling point temperature order ( $M$ $<E<B$ ) might be proportional to the response (y), it can be dealt with by the conference matrix. In another case, we obtained three types of lubricants (a, b, c) for machine cutting. Since the processing accuracy ( y ) was assumed to be proportional to the viscosity (V) of the lubricant, it was rearranged in the viscosity order (c> a> b) and assigned to the conference matrix.

### 3.5 Handling of the interaction of the conference matrix

Among the effects, the main effect is considered to be the largest, but in the study of complex systems, the interaction between factors strongly affects the prediction accuracy of the optimum conditions. For $\mathrm{L}_{8}, \mathrm{~L}_{16}$, etc., the interaction between two factors can be assigned to the orthogonal array inside, but it cannot be done with the conference matrix.

Therefore, in the conference matrix, the level of the allocation factor is changed for each experimental plan, and the interaction is understood with the factor tendency while improving the characteristics.

The specific procedure is (1) to identify the level difference between the coefficient graph and the best No (2). A new level will be adopted with the aim of improving the characteristics from the coefficient graph. Actually applied to this case. (1): From the coefficient graph in figure 3, the difference between the optimum CV A1B1C3D3 and the best No5: A1B1C3D2 is [D2, D3].: Adopt [D3] for the next experiment. (2): from the coefficient in figure 3, there is the outside levels of the original like [ $\mathrm{A} 1 \rightarrow \mathrm{~A} 0, \mathrm{~B} 1 \rightarrow \mathrm{~B} 0, \mathrm{C} 3 \rightarrow \mathrm{C} 4$ ] to improve CV characteristics.
$C_{4}\left(2^{1} 3^{3}\right)$ to which these are assigned is shown in table 16 , and the obtained results are shown in figure 6.

| Level | $\boldsymbol{\beta N 1}$ | $\boldsymbol{\beta N 0}$ | $\boldsymbol{\beta N 2}$ |
| :---: | :---: | :---: | :---: |
| A1 | 3.961 | 4.272 | 4.630 |
| A2 | 2.987 | 3.228 | 3.507 |
| A3 | 2.012 | 2.184 | 2.385 |
| B1 | 1.815 | 1.949 | 2.101 |
| B2 | 2.987 | 3.228 | 3.507 |
| B3 | 4.159 | 4.508 | 4.913 |
| C1 | 3.543 | 3.874 | 4.264 |
| C2 | 2.987 | 3.228 | 3.507 |
| C3 | 2.431 | 2.583 | 2.750 |
| D1 | 3.091 | 3.364 | 3.683 |
| D2 | 2.987 | 3.228 | 3.507 |
| D3 | 2.883 | 3.093 | 3.332 |

Table 16: Experimental design assigned to $\mathrm{C}_{4}$.


Figure 6: CV (Top): Average (Bottom) Relationship between $\mathrm{C}_{6}$ and $\mathrm{C}_{4}$.

In figure $6, C_{6}$ and $C_{4}$ are combined, and the interaction relationship can be understood from the relationship between the level change and the response (characteristic).

In the conference matrix, the interaction is understood by describing the factors and effects of each experiment on the same coefficient graph. We will visualize the interaction on figure"

## 4 Conclusion

The conventional parameter design applies the mixed type orthogonal array to the allocation and the logarithmic transformation of the SN ratio and sensitivity to the analysis. In this paper, we proposed the C matrix for allocation and the coefficient of variation and average for analysis.

The number of experiments was reduced to $1 / 3$ by applying the C matrix.

The allocation of classification factors, which was considered difficult, assumes a causal relationship between the response and the factors, and corresponds in order of the physicochemical constants.

The interaction effect of the C matrix without the interaction columns is confirmed by conducting an experiment incorporating the following (1) and (2) and describing it in the same coefficient graph.

1. Different levels between the optimum conditions of Coefficient graph the and the best condition of C matrices.
2. Outer level in the direction of improving the characteristics in the coefficient graph.

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