



## State Function of the Dynamic Drive System of Servo Systems Used in Myoelectric Prostheses

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### Abstract

Engineering's involvement in the sciences that affects people's lives has generated relatively new disciplines. Rehabilitation is the biomedical area with the greatest impact. This work seeks to represent the solution of the equation of state of the dynamics that interfere with each of the variables that contribute to the stable functioning of a motor plant that is used in myoelectric prostheses. The contribution of each one of them must contribute to the global stability of the dynamic system. Through Modern Control Theory, it is possible to face any problematic situation using state equations without considering mathematical rules used for recursive algorithms. When working in the state space using control tools, it is possible access to developments and analyzes that complement those carried out in the space of time and frequency. The simplification of the different developments will be reflected with new findings, giving value to new concepts such as the transition matrix and controllability. The quantization of some properties to perform the required movements is necessary for the manipulation of the control signal function to be implemented. The results obtained allowed us to discern and graph the trajectories in the state space of the intervening variables.

**Keywords:** Modelling; States; Control; Tmatrix

### Introduction

The contribution of biomedical engineering to the area of rehabilitation consists of the design of useful devices to automate the different therapies and give patients the necessary autonomy for a better development at the social level [1]. More than one billion people, or around 15% of the world's population, have some form of disability. The number of people with disabilities is increasing dramatically. This is due, inter alia, to demographic trends and the increase in the prevalence of chronic diseases.

Almost everyone is likely to experience some form of disability – temporary or permanent – at some point in their lives. People with disabilities have less access to health care services, so their care needs are often neglected.

If there are health services for people with disabilities, they are always of poor quality and do not have insufficient resources. There is an urgent need to expand services for persons with disabilities in primary health care, especially rehabilitation interventions [2].

It is essential to arrive at a useful model of application, finished the prosthesis for people with disabilities, evaluate cases related to an efficient model of energy management and present an optimal regulator of a servo motor designed to generate movements of the joint of a robotic arm in the rehabilitation of a patient [3].

The design of artificial limbs requires a complete knowledge not only of the mechanics of the mechanisms, but also a clear understanding of electromechanical devices, among which drive motors play a key role in the area of prostheses.

The maximum speed, strength and stability of the artificial anatomical limb are not yet comparable with the real (human) one.

These limitations are due to physical constraints of current technology to achieve the properties exhibited by the natural limb. Combining speed and muscle strength with a technological actuator is not an easy task, mainly when choosing a drive motor with the right speed-to-torque ratio [4]. The instability of the drive motors adds to the complexity of the proper design of the prostheses.

The specific kinematics of a mechanical arm studies the analytical description of the robot’s spatial displacement as a function of particular time, and the relationships between the position of the articulation variables and the orientation of the robot’s arm.

Direct kinematics tries to determine the position coordinates from the knowledge of the articulation variables, that is, from the angles they form. Cartesian coordinates depend on articular coordinates.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{f}(\theta_1, \theta_2, \theta_3)$$

Reverse kinematics seeks to determine the angles that each joint must form to bring the arm to a known desired position (or coordinates).

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \vec{f}^{-1}(x, y, z)$$

It is possible to try to choose some method that allows us to control the manipulator. This control tends to allow the end effector to be located in a certain position, and with a given orientation, in such a way as to perform some task.

To reach any point X of the accessible space of the manipulator, it means to locate the kinematic pairs, which in this type of robot are all rotating, in a given angular position.

If you want to move to another point X<sub>i</sub> of space, what you must do is calculate what will be the angle that all the joints should rotate. Since the solution of the problem is not unique, some kind of strategy must be used to make the movements, both in their se-

quence and in their value. It is unlikely that these movements will all be done together, but it is convenient that it takes place one at a time. The acceleration and speed of the movements must also be controlled, so as not to exceed certain maximum values.

In systems that require a lot of precision, direct current motors are used, which are of low power.

The objective of this work is to find the State Function of the servomotor that drives the motormotors of the manipulator arm.

### Materials and Methods

From here, we will work with the engine model that will be in charge of artificially performing the muscular actions of the elbow joint. The data for the servo motor simulation model were procured from a real motor (RE Maxon® 40-40 mm model) [5] frequently used to drive biomechanical prostheses [6].

This servomotor is incorporated in the elbow joint of the prosthesis and its function is only to give energy so that from the myographic signal received, the prosthesis is in charge of carrying out the extension, flexion and prono-supination movements.

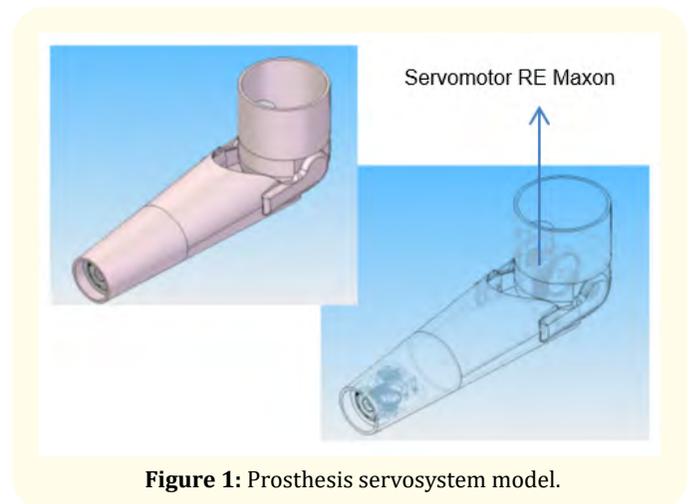


Figure 1: Prosthesis servosystem model.

### Motor model

The modern trend in bioengineering systems is toward greater complexity. Due to the need to meet increasingly demanding requirements in the behavior of control systems, the concept of the state arises.

The state of a dynamic system is the smallest set of variables (called state variables), since with the initial conditions of the phenomenon to be studied, they completely determine the behavior of the system.

Studies include other concepts such as state vector, state space and equations in state space [7].

This analysis will address the problem raised at the end of the Introduction making use of all these control tools.

As described by Alvarez Picaza, *et al.* [8], the following direct current motor is presented.

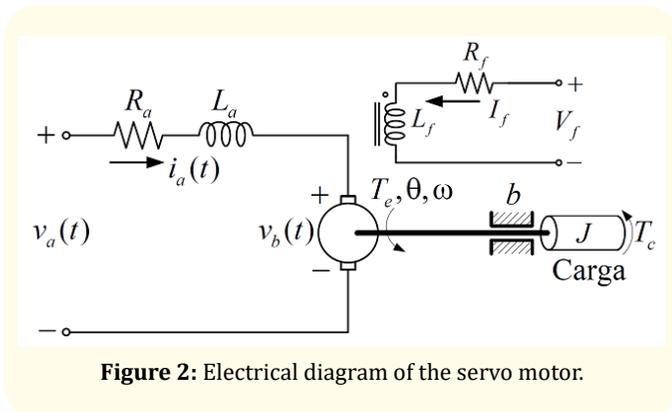


Figure 2: Electrical diagram of the servo motor.

The model to be considered drives a load through a rigid shaft. It is taken into account that the field current  $I_f$  or the field flow are constant, this machine can be controlled only by the voltage  $v_a(t)$  applied to the armature [9].

There is a relationship between the electric torque of torsion  $T_e(t)$  and the current in the armature  $i_a(t)$ , as indicated in equation (1). This ratio is the torque constant  $K_t$

$$T_e(t) = K_t i_a(t) \quad \text{----- (1)}$$

The angular velocity of the shaft is directly proportional to the voltage at motor terminals  $v_b(t)$ ,

$$v_b(t) = K_b \frac{d\theta(t)}{dt} \quad \text{----- (2)}$$

Where  $K_b$  is the motor velocity constant. The total moment of inertia of the charge is  $J$  and  $\theta$  the angular displacement,  $b$  is the viscous friction coefficient and  $T_c$  is the torque produced by the charge.

In the armature circuit, it is verified that,

$$v_a(t) = u(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta(t)}{dt} \quad \text{----- (3)}$$

Here  $R_a$  and  $L_a$  represent the impedance of the armature winding.

From equation (1),

$$K_t i_a(t) = J \frac{d^2\theta(t)}{dt^2} + b \frac{d\theta(t)}{dt} \quad \text{----- (4)}$$

We chose to work in the space of states, whose equations contain all the information of the internal dynamics of the system, allow to easily include the initial conditions and in general are of simple resolution. Figure 2 shows the servo motor model of in state space [10].

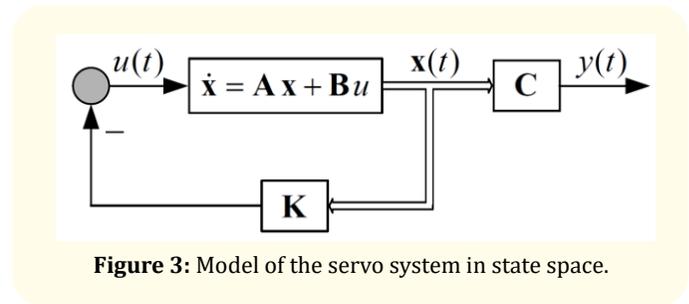


Figure 3: Model of the servo system in state space.

We choose as state variables of this system to,

$$\begin{aligned} x_1 &= \theta(t) = \omega(t) \quad \text{----- (5)} \\ x_2 &= i_a(t) \end{aligned}$$

Being  $x_1$  the angular velocity and  $x_2$  the armature current.

Indicates that,

$$\begin{aligned} \dot{x}_1 &= \dot{\omega} = \dot{\theta} \quad \text{----- (6)} \\ \dot{x}_2 &= \frac{di_a}{dt} \end{aligned}$$

From equation (4)

$$\dot{\omega} = -\frac{b}{J} \omega(t) + \frac{K_t}{J} i_a(t) \quad \text{----- (7)}$$

And from (3)

$$\frac{di_a(t)}{dt} = -\frac{K_b}{L_a}\omega(t) - \frac{R_a}{L_a}i_a(t) + \frac{1}{L_a}u(t) \tag{8}$$

In matrix way we have,

$$\begin{bmatrix} \dot{\omega}(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} u(t) \tag{9}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix}$$

**Transition matrix**

The matrix is called the Transition Of States Matrix and is denoted by  $e^{At}$ ,

$$\Phi(t) = e^{At} = (\mathbf{I} + \mathbf{A}t + \frac{1}{2!}\mathbf{A}^2t^2 + \frac{1}{3!}\mathbf{A}^3t^3 + \dots) \tag{10}$$

It governs the trajectories of states in a finite time interval t. It is the natural response of the system.

**Status function**

Let be a control system,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{11}$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

To know the response or output of the system for a given input and a set of given initial conditions, the solution of the Equation of State of the system must be found.

The solution  $\mathbf{x}(t)$  is sought for a system of type (11),

$$\mathbf{x} - \mathbf{A}\mathbf{x} = \mathbf{B}u \tag{12}$$

We wrote about the State Of The System Equation and analyzed it in the Space of Laplace.

$$s\mathbf{X}(s) - \mathbf{X}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \tag{13}$$

To finally,

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{X}(0) + (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}\mathbf{U}(s) \tag{14}$$

Anti-transforming the system we get,

$$\mathbf{x}(t) = e^{At} \mathbf{x}(0) + \int_0^t e^{A(t-\tau)} \mathbf{B}\mathbf{u}(\tau) d\tau \tag{15}$$

The solution is the sum of a term formed by the transition of the initial state (Natural Response) and a term that arises from the input vector (Forced Response).

**Controllability**

Broadly speaking, controllability studies the possibility of guiding or bringing the states of a system to a desired position by means of the input signal. The equation of state (7) or the pair [A,B] is said to be controllable if for any initial state  $\mathbf{x}(0) = \mathbf{x}_0$  and any final state  $\mathbf{x}_1$ , there exists an input u that transfers  $\mathbf{x}_0$  to  $\mathbf{x}_1$  in a finite time interval.

For the system to be controllable, it must be met that the determinant of the Controllability matrix is non-zero,

$$\det(\mathbf{C}) = \begin{vmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \mathbf{A}^2\mathbf{B}^2 & \dots & \mathbf{A}^{n-1}\mathbf{B} \end{vmatrix} \neq 0 \tag{16}$$

Or

$$\text{rank}(\mathbf{C}) = n \tag{17}$$

being  $\mathbf{C}$  the Controllability Matrix.

**Results**

The following values were taken from the manufacturer’s catalog:

$R_a$  = Armor resistance = 1.16Ω.

$L_a$  = Armature inductance = 0.329 mH.

$K_t$  = Constant the torque= 60.3 mNm/A.

$K_b$  = Speed constant = 158 rpm/V.

$b$  = Friction coefficient = 3.04 rpm/mNm.

$J$  = Moment of inertia = 138 gcm<sup>2</sup>.

From (11) we get the solution of the equation of state of the system,

$$\mathbf{x}(t) = \begin{bmatrix} a/b \cdot \exp(-3t) \cdot c^{(1/2)} \cdot \sin(1/d \cdot f^{(1/2)} \cdot t) \\ + \exp(-3t) \cdot \cos(1/g \cdot h^{(1/2)} \cdot t) \\ [j/m \cdot \exp(-3t) \cdot n^{(1/2)} \cdot \sin(1/q \cdot r^{(1/2)} \cdot t) \\ + \exp(-3t) \cdot \cos(1/v \cdot w^{(1/2)} \cdot t)] \end{bmatrix}$$

Numerically

$$\begin{aligned}
 \mathbf{x}(t) = & \\
 & [1,0188-0,0188*e^{-1,7739*t}]*\cos(14,3797*t)+0,0063*t \\
 & [0,9937*e^{-1,7739*t}]*\cos(14,3797*t)+0,0063
 \end{aligned}$$

From (15) we observe that the solution  $\mathbf{x}(t)$  is the sum of a term formed by the transition of the initial state (Natural Response) and a term that arises from the vector of inputs (Forced Response) [11].

The results found were worked with the MATLAB® program (Matrix Laboratory) [12] seeking to find the evolution of this function in the State Space.

The treatment of events in the space of states gives us the advantage of being able to observe behaviors that would not be visible in the space of times or frequencies.

There are internal dynamics within the systems that tend to grow indefinitely and others that make the system tend to a state of equilibrium [13].

For the operating data, it is sought to determine that the system is controllable (convergent function), which can be seen in the figures shown below.

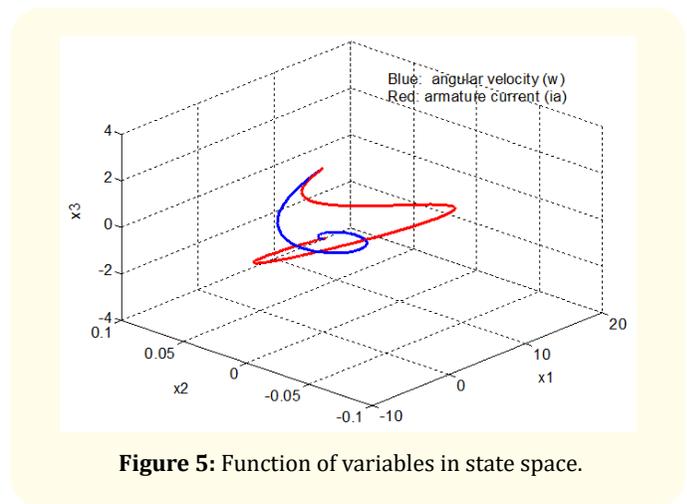


Figure 5: Function of variables in state space.

The transition of states [14] that governs the system (Figure 3) is given by.

$$\Phi(t) = \begin{bmatrix} -0,0207 & 0,0050 \\ -5,5005 & -0,0608 \end{bmatrix}$$

The corresponding surface diagram can be seen in figure 6.

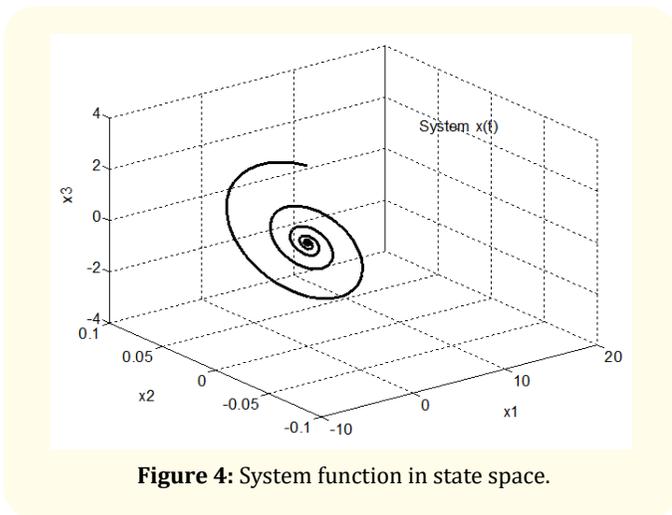


Figure 4: System function in state space.

Figure 4 shows the drive state function of the servo system, which is convergent which means that for these working parameters, the system is controllable.

Similarly, you can find the state functions for each of the variables as can be visualized in figure 5.

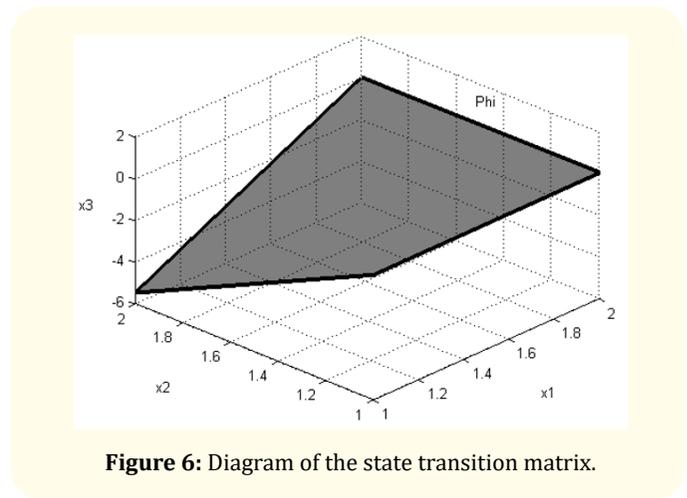


Figure 6: Diagram of the state transition matrix.

There is a compromise between the speed of the response and the sensitivity to disturbances and noise in the measurement.

That is, if the response speed is increased, the adverse effects of disturbances and noise are generally increased in the measurement.

## Conclusions

Working in the space of states making use of the tools of modern control theory allows to find new parameters of drive of the servo motor used in myoelectric prostheses, thus complementing with the other data that can be obtained assiduously from working in the space of time and frequencies.

In this space it is possible to make the graphs indicative of the stability of the system (Figure 4 and 5), otherwise the system would be like a black box inaccessible to analyze and graph. You can observe the evolution of the variables, state by state.

The tools used for the Modern Control Theory allow to carry out the pertinent mathematical models and obtain quantitative results without the need to use recursive algorithms.

The transition of states is an indicator of making the black box, a little clearer, it specifies the increase of the states during the process.

Both the different state functions and the transition matrix can be plotted and perform a detailed qualitative analysis of the intrinsic behavior of the dynamical system.

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