



Study and Comparison of the Electre I and MACUQ Methods for an Application in Life Cycle Analysis

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Abstract

Life cycle analysis is a method of assessing the potential environmental aspects and impacts related to the manufacture of a product, the management of waste or the installation of a refinery. It is used as part of an eco-concept approach and does not replace environmental standards. However, it provides a means for decision-makers to compare different possible solutions. Indeed it is a field of scientific research (study of impacts, weightings...). The life cycle analysis allows from an aggregation function to evaluate the different indicators and to make a classification, a sorting and to choose one or more possible solutions. This is also the role played by the analysis multi-criteria decision support. The objective is to apply the multi-criteria analysis to the life cycle analysis, by proposing aggregation methods to evaluate the alternatives on the basis of the potential criteria which sometimes are divergent. However among the existing methods, some have allowed to find a compromise in a situation of choice where no solution is perfect.

Keywords: Life Cycle Analysis; MultiCriteria Decision Analysis; Aggregation-Choice of Method

Introduction

Many qualitative and quantitative mathematical tools have been proposed in order to allow the decision-maker to make a good choice or to tend towards a better understanding: these are multi-criteria analysis methods for decision support. They have made it possible to solve economic and environmental problems with multiple and sometimes conflicting criteria.

At the environmental level, the assessment of the environmental impacts of refinery facilities, mining waste landfill and waste water treatment fall within the scope of life cycle analysis. It allows indicators to be combined and aggregated in order to propose solutions to multi-criteria single-decider problems or mul-

ti-decider single-criteria problems or multi-criteria multi-decider problems [8].

Life cycle analysis and multi-criteria analysis are peers. It uses the same methods as those of multi-criteria analysis to solve problems of choice, sorting and classification [9,10].

In the literature, many existing methods of multi-criteria analysis for decision support have been developed to analyze the environmental impact taking into account ecological standards [11,12].

For our work, we are going to analyze the environmental impact of installing a refinery in agglomeration with Electre I and MACUQ and then proceed to a comparative study in order to make a choice of method that meets the needs of the decision maker.

Literary Review

Method of Electre I

The ELECTRE I [1] method solves multi-criteria problems of choice. This method makes it possible to identify a subset of actions offering the best possible compromise. Often used in the selection of competing projects, in order to identify the best performing subset of projects based on the criteria considered. In the case of the Electre I method, we define true-criteria, we also find a notion of competition in this method; retain the best.

We consider a set A of m actions, which represent the object of the decision, the goal of which is to identify a subset of actions offering a better compromise among the starting set. We define for each criteria an evaluation function g_j (where $j = 1$ to n , n is the number of criteria), for each criteria, we evaluate a weight ω_j which increases with the importance of the criteria.

Consider two actions a and b

Concordance principle: A majority of criteria, taking into account their importance, must support aSb assertion (majority principle) is denoted $C(a,b)$ is between $[0,1]$.

It measures the relevance of the assertion a outranks b

Principle of non-discordance: Among the criteria that do not support assertion aSb, no one should express too strong a disagreement (principle of respect for minorities) [2-5].

The ELECTRE I method aims to obtain a partition of A into two subsets.

N and $A \setminus N$. N is called the kernel of the outranking graph (this is the seat of the non-outranking actions); the best action is contained in N , such that.

- Any kernel action does not override each other.
- Any action outside the core is outclassed by at least one action of N The procedure of the method is as follows:
 - Define the criteria j for judging actions (the criteria are true-criteria).
 - Assign to each criteria j a weight ω_j that is all the greater as the criteria is important.

- Calculate for each pair of actions (a_i, a_k) the discrepancy index as follows: with

$$C(a_i, a_k) = \frac{\sum_{\forall j, g_j(a_i) \geq g_j(a_k)} \omega_j}{K}$$

This index $C(a_i, a_k)$ measures the arguments of n in favor of the assertion a_i outclasses a_k .

The concordance index is therefore given by the sum of the weights of the criteria for which the action is at least equal to the action a_k on the set Criteria.

Calculate the discrepancy indices: $D(a_i, a_k) = 0$ si $\forall j, g_j(a_i) > g_j(a_k)$

$$D(a_i, a_k) = \frac{1}{E} \max_j [g_j(a_k) - g_j(a_i)] \dots\dots\dots(1)$$

Otherwise with E is the maximum difference between the same criteria for two given actions.

\hat{d} . The outranking relation for Electre I is constructed by comparing the concordance and discordance indices with limit thresholds of concordance c and discrepancy.

Thus, a outclasses b,

$$\text{if: } a_i S a_k \iff D(a_i, a_k) \text{ and } C(a_i, a_k)$$

The MACUQ method

The quadratic mean of n values x_i is obtained in the following way :

$$q = \sqrt{\frac{\sum x_i^2}{n}} \dots\dots\dots(2)$$

Presentation of the collective aggregation function based on the quadratic mean

This aggregation function is called the collective aggregation method using the quadratic mean (MACUQ). It is suitable for solving single-decider and multiple-criteria (SDMC) problems. In his modeling, the discrete alternatives are evaluated with regard to the different criteria in order to obtain an order of the alternatives.

Let A be a set of actions; Suppose $A = \{a_1, a_2, a_3, \dots, a_k, \dots\}$

$g_j(a)$ is the performance of a for the criteria j whose total number is m .

A G function is an additive value function if it has the form

$$G(a) = \sum_{j=1}^m k_j v_j [g_j(a)] ; k_j > 0$$

With $v_j [g_j]$ increasing monotone function of g_j ,

It is not restrictive to impose $0 \leq v_j [g_j] \leq 1$ and $\sum_{j=1}^m k_j = 1$

It should be noted that in our situation it is a maximization, which explains the growth of the functions. In the case of a minimization, the functions are decreasing. $g_j(a)$ is the performance of a for the criteria j whose total number is m .

A function G is a function of additive value if it of the form

$$G(a) = \sum_{j=1}^m k_j v_j [g_j(a)] ; k_j > 0$$

With $v_j [g_j]$ increasing monotone function of g_j ,

It is not restrictive to impose $0 \leq v_j [g_j] \leq 1$ and $\sum_{j=1}^m k_j = 1$.

It should be noted that in our situation it is a maximization, which explains the growth of the functions. In the case of a minimization, the functions are decreasing.

- Let D be the decider
- Let m be the number of criteria whose set of indices is $\{1,2,\dots,m\}$
- Let N be the number of actions and their set; $A = \{a_1, a_2, \dots, a_k, \dots, a_N\}$
- Let $\{v_1, \dots, v_m\}$ be the set of preferences that represents the decision maker evaluations of actions with respect to the criteria

Let G_i denote the additive value aggregation function for the alternative a_i .

The collective aggregation function using the quadratic mean is as follows,

$$U(a_k) = \sqrt{\frac{\sum_{k=1}^N G_k^2(a_k)}{N}} \quad \text{-----(3)}$$

With $G_k(a_k) = \frac{\sum_{j=1}^m \omega_j^k g_j^k(a_k)}{\sum_{j=1}^m \omega_j^k}$ ----- (4)

Et $k = 1, \dots, N; j = 1, \dots, m$

Example

The example [7] deals with the choice of a project, among six (6) competing projects, for the realization of a refinery. Each project is evaluated on the basis of five (5) environmental criteria

- C_1 : Noise pollution
- C_2 : Separation of territory
- C_3 : Air Pollution
- C_4 : Impact on land use planning
- C_5 : Impact on recreational activities

The importance of each criteria in decision-making is translated by a weight ω_j .

Each project is evaluated according to the criteria selected using a qualitative scale and scores. The higher the score, the lower the environmental impacts of the project. The performance chart is given in the following table.

Project	C_1	C_2	C_3	C_4	C_5
Weight	3	2	3	1	1
P_1	10	20	5	10	16
P_2	0	5	5	16	10
P_3	0	10	0	16	7
P_4	20	5	10	10	13
P_5	20	10	15	10	13
P_6	20	10	20	13	13

Table 1: Performance chart.

Application to the Electre I method

Calculate the concordance index

$$C(P_1, P_2) = \frac{3 + 2 + 3 + 0 + 1}{10} = 0.9$$

$$C(P_2, P_1) = \frac{0 + 0 + 3 + 0 + 1}{10} = 0.4$$

$$C(P_1, P_3) = \frac{3 + 2 + 3 + 0 + 1}{10} = 0.9$$

$$C(P_3, P_1) = \frac{0 + 0 + 0 + 1 + 0}{10} = 0.1$$

$$C(P_1, P_4) = \frac{0 + 2 + 0 + 1 + 1}{10} = 0.4$$

$$C(P_4, P_1) = \frac{3 + 0 + 3 + 1 + 0}{10} = 0.7$$

The other concordance indices are calculated in the same way and we obtain the following concordance matrix.

	P ₁	P ₂	P ₃	P ₄	P ₅	P
P ₁	-	0.9	0.9	0.4	0.4	0.3
P ₂	0.4	-	0.8	0.4	0.1	0.1
P ₃	0.1	0.6	-	0.3	0.3	0.3
P ₄	0.7	0.9	0.7	-	0.5	0.4
P ₅	0.7	0.9	0.9	1.0	-	0.6
P ₆	0.7	0.9	0.9	1.0	1.0	-

Table 2: Concordance Matrix.

Calculate the discrepancy index

The discrepancy index is calculated for a value of E = 20 - 0 = 20

$$D(P_1, P_2) = \frac{6}{20} = 0.30$$

$$D(P_2, P_1) = \frac{15}{20} = 0.75$$

$$D(P_1, P_3) = \frac{6}{20} = 0.30$$

$$D(P_3, P_1) = \frac{10}{20} = 0.50$$

$$D(P_1, P_4) = \frac{10}{20} = 0.50$$

$$D(P_4, P_1) = \frac{15}{20} = 0.75$$

The other discrepancy indices are calculated in the same way and we obtain the following discrepancy matrix:

Let's identify the projects with the least environmental impact at the matching threshold c = 1 and at the discordant threshold d = 0.

P₁P₂P₃ and P₆ are incomparable, on the other hand P₆SP₄, P₆SP₅ and P₅SP₄.

	P ₁	P ₂	P ₃	P ₄	P ₅	P
P ₁	-	0.3	0.30	0.50	0.5	0.75
P ₂	0.75	-	0.25	1	1	1
P ₃	0.5	0.25	-	1	1	1
P ₄	0.75	0.30	0.30	-	0.25	0.5
P ₅	0.5	0.30	0.30	0	-	0.25
P ₆	0.5	0.15	0.15	0	0	-

Table 3: Discrepancy Matrix.

The Projects to be retained are the set of elements of the subset N offering the best possible compromise.

Core N consists of N = {P₁, P₂, P₃, P₆}.

For thresholds c = 0.9 and d = 0.15 we obtain the following result: P₆SP₂, P₆SP₃, P₆SP₄ and P₆SP₅. Projects P₁ and P₆ are incomparable.

The Projects to be retained are the set of elements of the subset N offering the best possible compromise. The core N consists of N = {P₁, P₆}.

Application to the MACUQ method

$$G_k(a_i) = \frac{\sum_{j=1}^{j=m} \omega_j^k g_j^k(a_i)}{\sum_{j=1}^m \omega_j^k}$$

Calculate

$$G(P_1) = \frac{10 \times 3 + 20 \times 2 + 5 \times 3 + 10 \times 1 + 16 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{30 + 40 + 15 + 10 + 16}{10}$$

$$G(P_2) = \frac{0 \times 3 + 5 \times 2 + 5 \times 3 + 16 \times 1 + 10 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{0 + 10 + 15 + 16 + 10}{10}$$

$$G(P_3) = \frac{0 \times 3 + 10 \times 2 + 0 \times 3 + 16 \times 1 + 7 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{0 + 20 + 0 + 16 + 10}{10}$$

$$G(P_4) = \frac{20 \times 3 + 5 \times 2 + 10 \times 3 + 10 \times 1 + 13 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{60 + 10 + 30 + 10 + 13}{10}$$

$$G(P_5) = \frac{20 \times 3 + 10 \times 2 + 15 \times 3 + 10 \times 1 + 13 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{60 + 20 + 45 + 10 + 13}{10}$$

$$G(P_6) = \frac{20 \times 3 + 10 \times 2 + 20 \times 3 + 13 \times 1 + 13 \times 1}{3 + 2 + 3 + 1 + 1} = \frac{60 + 20 + 60 + 13 + 13}{10}$$

Calculate

$$U(a_k) = \sqrt{\frac{\sum_{k=1}^{k=N} G_k^2(a_k)}{N}}$$

$$U(P_1) = \sqrt{\frac{30^2 + 40^2 + 15^2 + 10^2 + 16^2}{10^2 \times 6}} = \sqrt{\frac{3081}{600}} = 2.26$$

$$U(P_2) = \sqrt{\frac{0^2 + 10^2 + 15^2 + 10^2 + 16^2}{10^2 \times 6}} = \sqrt{\frac{681}{600}} = 1.06$$

$$U(P_3) = \sqrt{\frac{756}{600}} = 1.12$$

$$U(P_4) = \sqrt{\frac{4869}{600}} = 2.84$$

$$U(P_5) = \sqrt{\frac{6294}{600}} = 3.23$$

$$U(P_6) = \sqrt{\frac{7938}{600}} = 3.63$$

The ranking of projects is done from best to worst project. Project P₆ would be the best project with less environmental impact.

Projects	Global Score	Ranking
P ₁	2.26	P ₆
P ₂	1.06	P ₅
P ₃	1.12	P ₄
P ₄	2.84	P ₁
P ₅	3.23	P ₃
P ₆	3.63	P ₂

Table 4: Overall score and ranking.

Comparison

The Electre I method has made it possible to have a lot of good projects that are incomparable between them. The result obtained is sensitive to the values of the thresholds such as the concordance threshold $c = 1$ and the discrepancy threshold $d = 0$ P₁ P₂ P₃; and P₆ are the good compromises.

At the thresholds of $c = 0.9$ and $d = 0.15$ we obtain two good projects P₁ and P₆.

All of these projects belong to the core according to the different assigned thresholds.

The MACUQ method allowed us to rank the alternatives and choose the P₆ project as the best of the projects. This is not the case with Electre I which gives a set of incomparable projects between them but does not allow them to be classified because all the good projects belong to the kernel. The MACUQ method offers more possibilities to the decision maker to classify the projects and choose the best understood.

Conclusion

The Electre I method has allowed us to have a batch of best projects that are incomparable between them. This does not make it easy for the decision maker to choose, who always remains in the decision-making embarrassment. Never mind, with the MACUQ method, we were able to make a ranking and choose a better project with a lower impact on the environment. From a descriptive point of view, the Electre 1 method and the MACUQ method make it possible to make a choice. They are easy to apply with fewer calculations to perform. On the other hand, choosing a better project, the MACUQ method would be advisable for the life cycle analysis.

The multi-criteria analysis is above all a tool allowing or helping the decision maker(s) to make a considered and structured choice. Assessing the environmental impacts with two multi-criteria methods, one compensatory and the other less compensatory, will allow us to have a broad view of the choices considered. The idea is to allow decision makers to have more choices to make. The obstacles of life cycle analysis are practical. For each problem, there is a method more suited to the problem than others. Choosing the most suitable method is not easy. We will continue the comparative analysis of MACUQ with the other methods of Electre and Promethee in order to give more possibilities in the choice of multi-criteria methods for the analysis of the life cycle.

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