

The Importance of Mathematical Education and the Role of Mathematics Teachers

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Abstract

The importance and role of teachers, as one of the key factors in the quality teaching process, have always attracted the attention of various authors and have been the subject of numerous theoretical considerations and empirical research, both in the field of pedagogy and beyond. Given the scope and breadth of problems in the teaching process, the role of teachers and ways of teaching are studied in different areas, such as, for example: school age of students, the level of student achievement, type of school, subjects, teaching methods, forms teaching work and the like.

In this paper, the role of mathematics teachers in relation to student success in this subject was studied. The research was conducted in order to determine the procedures of teachers who realize their professional role in the most important areas of teaching activities (organization and implementation of teaching; developing motivation and interest of students to learn mathematical content; achieving quality communication and interaction between teacher - student), and to determine their connection with the achieved level of student success in teaching mathematics.

Keywords: Role of Teachers; Subject Teaching of Mathematics; Planning, Preparation and Implementation of Teaching

Introduction

The term mathematics is in frequent use in various social spheres. Like subject, it activates logical thinking in students and thus contributes developing their competencies and thus enabling them for certain animals invitations and choosing future occupations. Mathematics teaches us to be wise and yes we solve problems step by step. It should be loved, because it is logical and tangled such is a challenge for many and a willingness to deal with it in different ways. Each of us can learn math. There is no human activity that does not depends on math, all high schools, and many colleges have math as school subject. The significance of the subject of mathematics is enormous, not only as an educational one subject, already contributes to personality building. Objectives and tasks of the subject are strictly defined, and in primary school they are intended for acquiring a basic language and mathematical literacy.

The pedagogical ability of the teacher and the climate he creates in the classroom are conditioned are characteristics of his

personality and can be crucial for the student's learning and course learning progress. Teacher behavior affects the image a student creates of him. At a younger school age, teacher behavior has the most influence. Personality the teacher leaves a "trace" with the students he teaches. The teacher must, u educationally, constantly be clear, understandable, honest, consistent, simple students.

A math teacher performs one of the most challenging and beautiful occupations. In addition, the mentioned occupation is very responsible. It depends on the teacher how much he will, and to what extent, be successful. The question is what are the qualities that will lead him to that way. This question led me to decide on this topic and its analysis will help me enable me to understand how important human and pedagogical characteristics are in order to achieved good results in teaching. In this paper, 12 traits are considered teachers as individuals and pedagogues. One teacher should be: competent, creative, characteristic, communicative, cooperative, cultural, constructive, computerized, broken (healthy), credible (confidential), cosmopolitan. All these the characteristics

are important so that the children's heads, which are needed, can be entrusted to him shape and steer on the right path. A teacher who will nurture these qualities, if any possesses, if not, then to make an effort to acquire them, and success in school will not be lacking. The goal diploma thesis is an explanation of each of these features and their impact on quality teaching. The listed characteristics are the unity of one successful teacher mathematics, but each is specific to itself and each took time, home schooling, a lot of upbringing and education, patience, because it is necessary to create from children, moral, creative people, with a lot of self-esteem, brought up, educated and with a positive view of the environment and the world.

Relationship between mathematics methodology and mathematics, pedagogy, psychology and didactics

Like any other science, the methodology of teaching mathematics is closely connected with other sciences and in the study of this subject the knowledge of those sciences is used. From the series scientific disciplines, whose knowledge he uses, for the methodology of teaching mathematics from special of importance and significance is pedagogy, didactics, and psychology. How mathematical upbringing and education includes the contents of mathematics by which they are realized laws of pedagogy and didactics according to which the government is governed, in accordance with student age, the specificity of the degree and quality of intellectual values of on which the methodology of teaching mathematics is based, in the study of one's own subject the help of other sciences is also needed, such as logic, philosophy, ethics, etc. It his is exactly what characterizes the interdisciplinarity of the methodology of teaching mathematics. Surely that the question of the connections and relations between the teaching methods is of great importance mathematics towards mathematics and pedagogy. It relies on knowledge and results mathematics and pedagogy, that some consider it mathematical and some pedagogical discipline. The methodology of teaching mathematics has established connections with mathematics and relations mostly through mathematical contents which are used for teaching. That is achieves the moment when the mathematical contents are put in the function of education and education.

Mathematics determines mathematical contents, and methodology determines conditions, forms and methods, i.e. ways in which their educational function will be realized.

The methodology of teaching mathematics establishes connections and relationships with mathematics and the path methodical interpretations of the contents determined by the mathematics teaching program. General pedagogy discovers, studies and determines the general laws of education, and methodology mathemat-

ics of the same law is studied in the teaching of mathematics. That connection of teaching mathematics with didactics is narrower, narrower and is reflected in taking over and using general didactic laws in teaching. This is because didactics is performed from the reality of many subjects and what is worth teaching in general, while the methodology of mathematics is valid only for that subject. Part of didactic and mathematical the content is common, and the part is different and is conditioned by the specifics of teaching mathematics. Thus, for example, the didactic rule of obviousness is the same for teaching all subjects, even for teaching mathematics. However, there is some content that condition the specificity of teaching mathematics, which is expressed in the applicable obviousness. The methodology of teaching mathematics is a mathematical and pedagogical discipline. For the methodology of teaching mathematics, its connection with psychology, and especially child, developmental, and pedagogical. The nature of mathematics. The contents point to the methodology of teaching mathematics in close cooperation with the psychological sciences. This is of special importance at the beginning of primary school, but also later.

The importance of mathematical education

Mathematics as a science is of great importance today, both in the lives of individuals and at the global social level (Bourdieu and Wacquant, 1992; Mackay, 2007). The emergence and development of mathematics was initially conditioned by practical life needs. In that earliest period, mathematical advances were used for various measurements and calculations in the fields of crafts, agriculture, construction, for calculating calendars and the like. With the development of civilization and literacy in all areas of social activity, mathematics, i.e. mathematical literacy, also developed, which enabled the further development of science and civilization (Božić, 2002). In modern trends of social development, mathematical knowledge is of great importance in all spheres and levels of human activities. According to some authors (Woodrow, 2003; Keitel, 2006), mathematics learning and mathematics education are today one of the key elements of society's economic prosperity.

Understanding the importance of mathematical knowledge in the modern world has contributed to the fact that in some developed countries the demands for the improvement of mathematics education are increasingly emphasized. Such demands can be seen even in the attitudes of the governments of certain countries or professional bodies, which insist on a larger number of workers with high knowledge in the field of mathematics (US Department of Education, 1997: 14, according to: Baranović, 2012: 166). Such requirements are also present in the European Research Consortium for Puzić and Informatics and Mathematics (ERCIM), emphasizing that the improvement of mathematics education is

an “absolute necessity” to ensure Europe’s economic competitiveness in the global market (ERCIM News, 2008). The importance of mathematical abilities and the need to develop them in education has been pointed out by other authors (Bell, 1999; Castells, 2000; Keitel, 2006; Mackay, 2007). In their opinion, the level of acquired knowledge in mathematics is the driving force of modern, scientific and technological development. Also, the mentioned authors believe that the social impact of mathematics is most noticeable in the field of information technology. This reasoning was explained by the accelerated flow of large amounts of information in various areas of social life (Mackay, 2007), in which the process of production and distribution of information goods and services is the basis of a new way production, based on mathematical knowledge. That direct consequence of the described changes is the application of mathematics in all key segments of society and, accordingly, mathematics is gaining more and more importance in the modern world (Bell, 1999; Castells, 2000). This understanding of the importance of mathematics is taken by Woodrow (2003), who believes that mathematics is a “language of science”, so in this sense, mathematics can be seen as a prerequisite for knowledge of technology.

Apart from the fact that mathematics influenced the development of modern science and information technology, its significance is also visible in the fact that at the same time it developed and perfected itself as a science. Namely, with the invention of computers, completely new mathematical disciplines were developed, such as programming mathematical and logical tasks, specialized computer languages, process modeling and others. Also, numerous mathematical theories and mathematical operations have emerged, the most famous of which are probability theory, information and control theory, game theory and the like. Today, there is almost no science that does not use mathematical achievements in the study of its subject. According to certain authors, mathematics is also used in those sciences in which it was not used before or its significance was negligible. Some of these sciences are, for example, biology, geology, linguistics, sociology, medical and other sciences (Charanchi, 2011: 17; Kaščelan, 2001, 48). When it comes to medicine or biology, mathematical advances are used in numerous studies in the fields of genetics, inheritance, maturation, metabolism, fatigue and the like. However, that is not all. Mathematics today occupies a significant place in the social sciences, which was not the case before. Based on different mathematical operations, using statistics, as well as in other ways, different quantitative and qualitative analyzes of social reality are performed, ie problems that are studied in the context of different social phenomena and relationships (Ćebić, 2000, 58). Mathematics, therefore, has become a part of life and culture, it has become a science without which life in the modern world is difficult to imagine.

The presented views of various authors provide insight into the fact that quality mathematics education provides students with significant advantages in the choice of future professional or academic activities. In addition, various authors believe that investing in the education of students in the field of mathematics potentially ensures the economic security and prosperity of an entire society. A brief account of the importance of mathematics education clearly indicates the fact that it is achievable higher levels of student achievement in mathematics need to create appropriate conditions. In this regard, the question of the quality of mathematics teaching as well as the role of teachers in this context is justifiably raised. Also, given the results of various studies (Aunola, *et al.* 2013; Pavlović-Babić and Baucal, 2013; Hidi 2000; Zimmerman and Kitsantas, 1999; TIMSS, 2003/2007, according to: Gašić-Pavišić and Stanković, 2012), Based on which it was determined that older students in primary school achieve significantly lower results and have more difficulties in learning mathematics compared to younger students, it is important to focus this issue on the role of teachers in subject teaching mathematics.

Through various activities carried out by the mathematics teacher in the teaching work, it is possible to significantly improve the quality of teaching, as well as student success (Antonijević, 2011; Kilpatrick and Swafford, 2002; Romano, 2009; Frenzel, *et al.* 2009; Hidi and Harackiewicz, 2000; Windsor, 2010). One of the important conditions for such a thing is the proper realization of the professional role of teachers in the areas explained in the previous part of the text (organization, planning, preparation and implementation of teaching; motivation and interests of students; quality communication and interaction in the classroom), which will be presented in more detail. chapters of this paper. In addition, in order to achieve appropriate teaching procedures, it is necessary to have a good knowledge of mathematics as a science, or mathematics as a teaching area.

Mathematics as a science and a subject

Proper organization and implementation of mathematics teaching requires from teachers a good knowledge of mathematics and mathematical relations in a broader context. This requirement refers to a good knowledge of important areas in mathematics as a science, complete knowledge of mathematics as a subject, understanding of the relationship between mathematics as a science - teaching mathematics, as well as respect for significant characteristics and specifics of mathematics teaching. The first of the questions that is asked in this context is: what is mathematics, i.e. what is the subject of its study? The answer to this question is not easy to give. Namely, even today there is no single and generally accepted definition of mathematics. One of the reasons for its

non-existence is the large number of different criteria according to which the authors have defined mathematics as a science. Thus, D’Alamber and Grassmann (according to: Špijunović and Maričić, 2016: 45), starting from the original meaning of the term “mathematics” (lat. *Mathema* - the science of quantities), defined mathematics as the science of the properties of quantities according to the subject, i.e. as a science on the relations between sizes and spatial forms. This definition of mathematics implied real quantities, i.e. quantities and their relations in the real world. However, the fact is that mathematics observes quantities in an abstract sense as well. No other science has performed such an abstraction of qualitative properties as is the case in mathematics, which has led to the existence of various definitions, according to which mathematics is defined as an abstract science. Based on that, in the literature there is a view that mathematics is a science that studies quantitative relations and spatial forms, which in the most direct sense of the word, are abstracted from reality (according to: Špijunović and Maričić, 2016: 46), i.e. that mathematics is a science that studies abstract concepts, created by generalizing the concepts of the real world (Vuković, 1996). Dejić and Mihajlović [1] explain the problem of precise definition of mathematics, as well as the final determination of its subject of study, by the development process of mathematics as a science.

These authors believe that the development of mathematics has changed its subject of study. For example, in the mathematics of Ancient Egypt, China, Babylon and India, the emphasis was placed exclusively on arithmetic, so for that period it would be correct to define mathematics as the science of numbers. On the other hand, the ancient Greeks mostly dealt with geometry, so they also examined numbers geometrically. In their time, it could be said that mathematics was understood as the science of numbers and geometric shapes. With the advent of Descartes’ analytical geometry, as well as Newton’s and Leibniz’s differential and integral calculus, the nature and subject of mathematics changed. The basis of this mathematics was movement and change, which marked a period of variables. In that sense, mathematics could be defined as a science that studies number, shape, movement, change and space [1].

Given the developmental characteristics of mathematics as a science, it can best be understood in a broader context, as the science of quantity (arithmetic); structures (algebra); space (geometry) and change (analysis). However, we should not lose sight of the fact that mathematics is a science that is constantly evolving and in which new mathematical disciplines are being re-created. At the beginning and during the 20th century, many abstract mathematical theories appeared, such as algebraic structures, topological

structures, algorithm theory, programming and others. This means that in mathematics nothing is given forever, but that one follows from the other, that it always strives for permanent development, so that no definition of mathematics should be considered final.

The relationship between mathematics as a science and mathematics as a teaching area is interdependent. On the one hand, the teaching of mathematics has always followed the development of mathematical scientific achievements. One of its basic tasks was to adapt the most important of what has been achieved in mathematics as a science to teaching, i.e. to students, so that mathematics teaching fully expresses the needs of society (Ćebić, 2000, 604). On the other hand, in order to achieve significant mathematical achievements, as well as their successful application, it is necessary to design and implement quality mathematics education (Špijunović and Maričić, 2016, 47). Therefore, it can be said that mathematics, as a subject, in a way represents a link between students and mathematics as a science. This interpretation of mathematics teaching can be derived on the basis of the previously mentioned views of the author on the importance of student education in the field of mathematics. Given the dynamics of the development of science and technology, nowadays mathematics education is becoming increasingly important, and the teaching of mathematics plays a key role in training students to practice mathematics. In this regard, Antonijević (2011: 86) concludes that any individual can successfully monitor the development of society and become an active participant in a particular field of work, if he is allowed to institutional education builds the foundations of the system of mathematical knowledge and develops a mathematical way of thinking.

In the basic education system, mathematics as a subject occupies a significant place. Acquisition of mathematical concepts and acquisition of knowledge in mathematics create the basis necessary for successful mastering of teaching materials in other subjects, such as physics, chemistry, technical education, informatics and others. In addition, the teaching of mathematics has a broader significance than the learning and cognition that takes place in it. This significance implies the influence that is exerted in the direction of the development of the personality of each student, in the intellectual, aesthetic or other aspects of the development of the personality.

Based on the educational plan, i.e. on the basis of the fund of classes, it can also be noticed that in our primary education, mathematics, as a subject (along with the Serbian language), occupies a significant place. According to the rules on the norm of hours direct work with students (Sl.glasnik.RS. Prosvetni glasnik, no.2/92 and

2/2000) in classroom teaching (from 1st to 4th grade) mathematics is represented by 5 lessons per week, while in subject teaching (from 5th to 8th grade) the fund of mathematics classes is 4 hours per week. Except in the fifth grade, where the teaching of the Serbian language is represented by 5 lessons per week, in other grades the teaching of mathematics, together with the teaching of the Serbian language, has the largest fund of lessons.

The presented data speak in favor of the fact that mathematics is an important subject in our schools. However, based on the above data, it is not possible to determine the quality of teaching hours, in terms of mastering mathematical requirements. That quality of teaching largely depends on the teachers and on the ways in which they organize teaching activities within the specified fund of classes. If mathematics teachers find appropriate ways to enable all students, often in their class of about thirty, to master the intended tasks and contents, then four hours a week can be considered sufficient time for students to achieve a high level of achievement in mathematics. However, if the situation is different, i.e. if teachers do not find appropriate ways to raise the quality of teaching to a higher level, which in addition to teacher training, largely depends on the number of students and class structure, then the question remains whether a fund of classes is provided sufficient for mastering the teaching material and achieving a higher level of student success in this subject. Therefore, based on the mentioned assumptions, the need for greater involvement of mathematics teachers, as well as for a deeper study of the role of teachers in mathematics teaching can be clearly seen. In addition to the above, these assumptions speak in favor of the complexity of the teaching process and for they entail a number of other issues, which go beyond the scope of this paper and which, in addition to the importance of studying the role of teachers in the teaching process, impose the need for deeper study of various contextual factors of the teaching or educational process.

Goals and tasks of teaching mathematics in subject teaching

The teaching of mathematics in primary school is determined by the contents, goals and tasks that correspond to the general age abilities of students. According to the rulebook on the curriculum for the seventh grade of primary education (Official Gazette of RS-Education Gazette "No. 6/2009, 3/2011; Rulebook 8/2013), the objectives of teaching mathematics include the following: acquiring basic language and mathematical literacy and progress of students towards the realization of appropriate standards of educational achievements; training students to solve problems and tasks in new and unknown situations; training to express and explain one's opinion as well as to participate in discussions with others; developing motivation for learning and interest in the sub-

ject contents; acquisition of elementary knowledge necessary for understanding phenomena and laws in nature and society; training students to apply the acquired mathematical knowledge in solving various tasks in life practice, training students to successfully continue mathematics education and self-education; as well as the contribution to the development of mental abilities, the formation of a scientific view of the world and the all-round development of the student's personality.

By the same rulebook, the goal of teaching mathematics is operationalized through a number of general and operational tasks of teaching mathematics. The general tasks of teaching mathematics are as follows: creating various opportunities to fully realize the purpose, goals and tasks of education, as well as the goals of teaching mathematics through various contents and forms of work during the teaching of mathematics; acquiring the knowledge necessary for understanding quantitative and spatial relations and laws in various phenomena in nature, society and everyday life; acquisition of basic mathematical culture, necessary for understanding the role and application of mathematics in various fields of human activity (mathematical modeling) for successful continuation of education and involvement in work; developing students' ability to observe, observe, as well as logical, critical, analytical and abstract thinking; developing students' cultural, work, ethical and aesthetic habits, as well as arousing mathematical curiosity; acquiring the ability to express oneself in mathematical language, clarity and precision of expression in written and oral form; adoption of basic facts about sets, relations and mappings; mastering basic operations with natural, integer, rational and real numbers, as well as adoption of the basic features of these operations; getting to know the most important geometric objects: lines, figures and bodies, and understanding their mutual relations; training students for precision in measurement, drawing and geometric constructions; preparing students to understand the relevant contents of natural and technical sciences; building positive traits of the student's personality, such as: systematicity, persistence, accuracy, orderliness, objectivity, self-control and a sense of independent work; acquiring habits and skills in using different sources of knowledge.

Based on the general tasks, the mentioned rulebook also determines specific tasks, which arise from the content of teaching for the seventh grade, and they provide training of students for the following: to understand the concept of square of rational number and square root; understand real numbers as measures of length, i.e. as points on the number line determined by the lines that represent such a measure; get acquainted with the concept of degrees and operations with degrees; are able to perform basic arithmetic operations with polynomials, as well as other identical transfor-

mations of these expressions (indicated in the program); get acquainted with the rectangular coordinate system and its application; are well acquainted with direct and inverse proportionality and practical applications; they know Pythagoras' theorem and are able to apply it to all studied geometric figures in which a right triangle can be observed; know the most important properties of polygons and circles; are able to construct individual regular polygons (with 3, 4, 6, 8 and 12 pages) and to draw other regular polygons by calculating the central angle and transmitting it with a protractor; know the most important patterns regarding polygons and circles and be able to apply them in appropriate tasks; understand the notion of scale length and properties of proportion; are able to translate into mathematical language and solve simpler textual problems; use elements of deductive reasoning (and derive simpler evidence within the studied content).

Insight into the goals and tasks of teaching mathematics, presented on the basis of the rulebook on the curriculum for the seventh grade, can be seen the importance and need for learning mathematics in modern society. These goals present the general requirements for training students in mastering mathematical content, as well as the requirements for developing mathematical thinking, which provides students with better opportunities for active participation in modern society. Also, the insight into the above clearly shows the importance that teachers, as one of the key factors in the educational process, have in teaching with older students of primary school.

With a deductive approach, from general to individual, in the above rulebook, based on the established goals, specific tasks in mathematics teaching are defined, whose analysis reveals certain details, ie dilemmas, which need to be addressed from the point of view of this paper. Namely, the first in a series of established tasks can serve as an example, i.e. "enabling students to understand the concept of the square of a rational number and the square root". The mentioned task explains what the students should be trained for, but it is not stated in which ways and procedures in teaching the mentioned task should be accomplished. Certainly, it is clear that there are no established "recipes" on the basis of which teachers should train and teach each student individually, nor is it possible to determine this within one teaching task in the rulebook, or even any other educational document. Therefore, teachers are expected to find the best ways to implement these tasks, which would achieve the set goals.

In this context, it is necessary to point out certain dilemmas, such as whether all mathematics teachers can understand the requirements before them, how they understand or accept their role

in solving them, whether all teachers can find adequate ways. to solve the above tasks and the like. In addition, the question is to what extent it is possible to accomplish these tasks, given the fact that in one class, teaching mathematics is attended by a large number of students with different psychophysical abilities, different interests and different levels of previously acquired mathematical knowledge.

These dilemmas are just some of the many, to which it is not easy to give precise answers, which requires additional research in the field of studying problems in teaching mathematics. However, the fact is that the mentioned requirements, i.e. tasks in the teaching of mathematics require good training of teachers for the realization of their professional activities. This implies that mathematics teachers must know their subject well, i.e. that their knowledge in the field of mathematics exceeds the requirements stated in the goals and tasks of teaching. Also, mathematics teachers must find adequate ways to implement the presented requirements in teaching. In order to be able to realize them, it is necessary for them to know their students well, their abilities and limitations, as well as to understand their problems and needs and, accordingly, to achieve quality cooperation with their students.

Specifics of mathematics teaching and the professional role of teachers

The teaching of mathematics contains a number of specifics, which require teachers to be fully committed and engaged in all areas of teaching. Some of the most important specifics of mathematics teaching are the following: mathematical abstractness, i.e. the abstractness of mathematical contents; the language of mathematics; developing mathematical thinking; high level of student activity in teaching mathematics; and the necessary continuity, gradualness and systematicity in the adoption of mathematical content.

Mathematical abstractness - abstractness of mathematical contents. One of the specifics, which is most often mentioned in the literature, is mathematical abstractness, i.e. the abstractness of mathematical contents (Bogutovac, 2008; Charanchi, 2011; Frenzel, *et al.* 2006; Ćebić, 200, Romano, 2009). In the etymological sense, the term "abstract" means a certain phenomenon or object that is sensually invisible and separate from the immediate, concrete reality, while "abstraction" means the thought process of obtaining what is abstract, or imagined or general, which is achieved by separation essential (essential) of irrelevant characteristics of certain objects, phenomena, quality and the like (Trebješanin, 2011: 40). In this process, abstract thinking is developed, which goes beyond the perceptual level of thinking and concrete phenomena, but already includes thought operations with abstract, general phenomena and symbols.

Mathematics is an abstract science and there are many abstract concepts in it, such as: number, point, line, plane, circle, triangle, figure, square root, polynomial, coordinate system and others. Therefore, one of the tasks of the teacher is to train students to understand abstract mathematical concepts, as well as their inter-relationships. However, this requirement is not easy to meet, for which, for example, the definition of mathematics as a science can serve. Thus, when it comes to definitions of mathematics as a science, it can be seen that its essence is often explained by terms that are more difficult to understand than the very term “mathematics” (Bogutovac, 2008: 70). This approach in mathematics teaching, where teachers explain abstract mathematical concepts with certain concepts that are not sure that all students understand them, can create additional problems in understanding and mastering mathematical content. In such situations, mathematics seems even more distant to students, which creates a feeling of confusion and misunderstanding, and can cause a decline in their interest in mathematics, the emergence of mathematical anxiety and the like (Schwartz, 2000; Hidi, 2006; Frenzel, *et al.* 2010), which certainly affects the level of student achievement in this subject.

When the age or cognitive conditions of students do not allow a certain mathematical concept to be treated as adopted by science, then the concept of this concept in school conditions can be simplified. However, this does not mean that in order to simplify mathematical concepts, certain features are added to them that are not in line with scientific meaning. In other words, abstract concepts in mathematics teaching should correspond to adequate concepts in mathematics as a science and there should be no discrepancies between them.

The problems of mathematical abstractness, i.e. the abstractness of mathematical content in teaching, were also discussed by Naziev (2015). This author points out that abstract objects and concepts appear not only in mathematics, but also in other sciences and subjects, as is the case, for example, in the teaching of physics (when considering atomic nucleus models), or in zoology (when speaks of animal classes) or in some other sciences (Naziev, 2015: 3). As one of the adequate ways to reduce mathematical abstraction, which would bring the teaching of mathematics closer to students, Naziev suggests considering (explaining) mathematical abstract concepts with continuous comparison with things and phenomena in the real world. Thus, this author accepts the view that mathematics is an abstract science, but does not ignore the fact that it arose from the practical needs of people, which, in his opinion, means that mathematics does not go beyond its abstractions, and that abstract concepts in mathematics must be explained by concrete concepts, ie concepts from the real world. What is also

important to note is that in the teaching of mathematics we often start from abstract concepts, which cannot be explained by concrete concepts or examples from the immediate reality (Hidi, 2006; Naziev, 2015). Then, previously abstracted concepts are used as a starting material for abstraction and idealization in mathematics. Such procedures form “abstractions from abstractions” in which, at first glance, it is difficult to notice the quantitative relations and spatial forms of the surrounding reality, but which nevertheless have their origin of them (Naziev, 2015: 4). So, although the concepts in mathematics are abstract, they still do not represent absolute thought constructions, but are connected with reality.

Therefore, the task of the teacher is to find appropriate didactic and methodological ways, by which they will perform the transformation of abstract mathematical concepts into concepts that will be understandable to the student. The role of mathematics teachers is reflected in the fact that, in addition to good knowledge of mathematics as a science and knowledge of teaching in general, in order to reduce the level of its abstractness should know theories of children’s thinking, to teach mathematics in accordance with certain didactic principles (Spijunovic and Maricic, 2016), as well as to harmonize the teaching of mathematics with modern tendencies of education, modern methods and means of learning (Schwartz, 2000; Frenzel, *et al.* 2010; Naziev, 2015).

The language of mathematics

In addition to being abstract, mathematics is specific in that it has its own mathematical language (Barwell, 2013; Fagan, 2005; Kilpatrick and Swafford, 2002; Kurnik, 2006; Naziev, 2015; O’Halloran, 2005). The fact is that language has a significant function in all sciences. It represents one of the four constitutive elements of every science, because it contains concepts and terms characteristic of a given science. When it comes to mathematics, the specificity of language is reflected in the fact that mathematics, much more than spoken language, uses different symbols and symbols, which represent mathematical concepts. Signs, symbols and signs in mathematics are of a universal character and their knowledge enables everyone to understand mathematical contents. The function of mathematical language in the teaching of mathematics can best be seen in the previously mentioned mathematical abstraction. Mathematical concepts and objects of study are mostly abstract and the fact is that they are difficult to understand or explain without the proper use of mathematical language. In doing so, it is important to point out that the nature of mathematics as a science requires mathematics teachers to be strictly disciplined in the use of mathematical language. Otherwise, there is a misunderstanding of mathematical content, and thus less progress in learning.

Knowledge of mathematical language is also important from the point of view of quality communication and interaction in mathematics teaching, as indicated by various authors (Yunus, Oktay and Isik, 2004; Kurnik, 2006). According to some authors, one of the basic tasks in mathematics teaching is to develop quality teacher-student communication (Kurnik, 2006), and in addition, understanding mathematical language has a broader significance, such as communication in social interactions (Morgan, *et al.* 2014). Kurnik (2006: 99), for example, distinguishes mathematically from spoken language and considers that mathematical language created by perfecting certain characteristics of spoken language, such as the breadth of its expressive possibilities (use of synonyms) and ambiguity (use of homonyms). When it comes to homonyms, Kurnik thinks that their use in teaching mathematics should be avoided, because it can lead to many ambiguities and misunderstandings. This author illustrated his position with an example with the term “root”, which in spoken as well as in mathematical language has multiple meanings (tooth root, root, plants, nerve root, square root, equation root, n^{th} root). On the other hand, the use of synonyms in mathematics is positive, because it simultaneously indicates the richness of language, but also more specifically defines a given concept, which can be seen in terms such as: square, regular quadrilateral, rhombus with right angle and the like. Similarly, the “richness” of mathematical language has been reported by other authors (Barwell, 2013; Morgan, *et al.* 2014; Yunus, Oktay and Isik, 2004). They also believe that there are many symbols and signs in mathematics, but also that these symbols allow for precise mathematical expression. It is therefore up to the teacher to teach students how to use mathematical language. It is not enough for students to use the language of mathematics, i.e. mathematical symbols and concepts only to denote certain phenomena and objects. It is necessary to understand the internal structure of language, as well as to understand the relationship between certain phenomena and objects.

The task of teachers in mathematics teaching is, therefore, to teach students to communicate mathematically, as well as to develop and nurture mathematical language in them. In addition to the adoption of mathematical signs, symbols and symbols (constant, number, unknown, variable, etc.), the role of the teacher is reflected in teaching students the proper use of mathematical language. Good knowledge of mathematical language and quality mathematical communication are one of the conditions for achieving a higher level of student success in mathematics teaching.

Necessary development of mathematical thinking necessary for learning mathematics. The development of mathematical thinking is another specificity that occupies a significant place in the teach-

ing of mathematics (Blanton, 2008; Cvetković, 1981; Carpenter and Levi, 2000; Prvanović, 1970; Usiskin, 1997; Windsor, 2010). The term “mathematical thinking” is quite complex and has not yet been precisely defined in the literature. Therefore, in various relevant works, other terms can often be found, such as: critical, logical, abstract or creative thinking (Kurnik, 2006; Maričić, 2006; Prescott, 2001; Schwartz, 2000). However, it can be considered that the term “mathematical thinking” basically encompasses all the mentioned terms (Maričić, 2006).

In the broadest sense, mathematical thinking implies logical thinking, which builds mathematical concepts, phenomena and processes, notices similarities and differences between them, reveals their mutual relations and dependencies (operated by them) and based on the above, make certain decisions. According to Cvetković (1981), one of the basic tasks of teaching mathematics is to enable students to look at the world from a mathematical point of view, which can be achieved by training students to adopt and connect mathematical concepts. This author believes that the process of adopting concepts is one of the central problems in the teaching of mathematics, as well as that the adopted mathematical concepts are the basic means of mathematical thinking (Cvetković, 1981, 69). Prvanović (1970) also dealt with the problems of mathematical thinking. Understanding that mathematical thinking constructs concepts and reveals mathematical relations and dependencies between them, and that in this way mathematical truths (facts) are revealed, Prvanovic considers it general and incomplete. One of the basic tasks of teaching mathematics, according to Prvanović, is to enable students to think mathematically and to know the features of modern mathematical thinking, which is enabled by investing in modern mathematics, i.e. creating teaching content aimed at developing students’ thinking operations (Prvanović, 1970: 14). According to this author, the role of teachers in the development of mathematical thinking in students is reflected in the use of different methods of problem-based learning, where the student should be placed in a situation where he will notice and distinguish what is mathematical. The teacher’s task is to guide the student through mathematical situations so that he himself realizes that different situations can be called by the same name, which allows the student progressive training to structure certain knowledge and use them in different situations.

The importance of mathematical thinking in their works is also pointed out by other authors (Prescott, 2000; Schwartz, 2000; Farner and Dafi, 2002; Kieren, 2004). Although they use the term “critical thinking” more often in their work, Prescott and Schwartz (Prescott, 2001, Schwartz, 2000), for example, consider that one of the basic tasks of mathematics teachers is to enable their students

to think mathematically. Prescott even thinks so that training students to think critically (mathematically) is as important as training them to calculate and solve mathematical problems (Prescott, 2001: 26). One of the ways to encourage students to develop mathematical thinking, these authors see in encouraging students to voice their answers, suggestions and ideas, as well as to participate in discussions with the teacher and other students. Kurnik (2006: 101) writes in a similar way about the importance of mathematical thinking, who believes that mathematical thinking is the basis for the realization of one of the basic didactic principles, ie the principle of permanence of knowledge. Many authors see the precondition for the development of mathematical thinking in students in the development of mathematical language (Barwell, 2013; Morgan., *et al.* 2014; Kurnik, 2006; Naziev, 2015), which indicates the interrelationship of these specifics of mathematics teaching, as well as the importance of engagement. teachers in all areas of the teaching process.

In the literature, in addition to the above terms, which include mathematical thinking, there are also terms such as “algebraic thinking” or “algebraic reasoning” (Carpenter and Levi, 2000; Greenes and Findell, 1998; Kieren, 2004). However, in a similar way as mathematical thinking, or critical thinking in mathematics, “algebraic thinking” means the development of thinking that includes analyzing the relationship between certain quantities, highlighting structures, noticing and analyzing changes, generalization, problem solving, estimation, inductive and deductive inference and others (Greenes and Findell, 1998; Kieren, 2004).

The problem of enabling students to think mathematically, in addition to the specific tasks that teachers need to fulfill in the implementation of their professional activities, requires teachers to know well the characteristics of the mental structures of their students, as well as their emotional state. Only with a good, i.e. complete knowledge of his students, as well as an adequate approach to each individual student, the teacher will be able to help students in appropriate ways in developing mathematical thinking.

High level of student activity in mathematics teaching. The importance of students’ active participation in teaching is often emphasized in the pedagogical literature. Whether it is active teaching, active learning, quality school (Ivić, Pešikan and Antić, 2001; Suzić, 2010; Glaser, 1994; Matijević, 2010; Azuka, 2013), the essence is that the emphasis is on greater engagement students in the teaching process. Most often, active teaching presents it as a teaching concept that is the opposite of the traditional school. The differences between traditional and active teaching can be briefly described in the following. Namely, traditional school (teaching) is

characterized by pre-defined plans and programs, teaching (monologue) method of teaching, student as a passive listener, reproduction of memorized material and the like. On the other hand, when it comes to active teaching, it is more focused on the student and his personality. In active teaching, teaching materials should be well designed and adapted to the interests of students, teaching methods should be dominated by practical and work activities, laboratory exercises, social activities, research tasks and more (Matijević, 2010; Prenzel 1992; Suzić, 2005). This view of active teaching applies to all subjects, not just mathematics, so the question can be asked, what is the difference between student activity in mathematics teaching compared to other subjects and why a high level of activity is seen as one of the specifics of teaching math? There are several answers to this question, and many of them are presented in the requirements of other specifics of mathematics teaching. As stated earlier, mathematics is an abstract science that has its own language dominated by numbers, signs, and symbols. For successful mastering of mathematical contents, it is necessary to know the language of mathematics which also enables the development of mathematical thinking. These specifics require students to actively participate in teaching mathematics, which includes solving problems, finding different solutions, exchanging opinions, asking questions, giving answers, presenting their own ideas and the like. In addition to the above, the answers to the question should be sought in another specificity of mathematics teaching, which is not presented in the previous part of the text. It is about the necessary continuity, gradualness and systematicity in the adoption of mathematical content. “Learning mathematics and mastering mathematical content is an upgrading process” (Schwartz, 2000: 63).

Unlike other subjects (Serbian language, biology, history, etc.), in the teaching of mathematics, the successful mastering of new teaching contents largely depends on how well the student is trained and how well he has mastered the previous teaching material. Each new step in mathematics is based on the previous one. New knowledge is upgraded to already adopted, and the application of acquired knowledge is used to determine them and create conditions for the adoption of new knowledge. The contents adopted in such a process are the basis for solving new problems and mathematical problems. Successful mastering of new requirements does not end the process of acquiring knowledge, but only begins, because then it is the turn of active learning of new teaching contents and so on. This process is one of the reasons why Dejić [2] believes that in mathematics classes should be dominated by heuristic conversation and discussion and not teaching, or presentation of teaching content and explanation by teachers. The teaching contents of some other subjects can be learned by multiple reading (repetition of reading), while with mathematics this is not the case.

Learning mathematics requires continuous work, as well as finding different ways to successfully solve problems, which requires a higher level of student activity. Some authors believe that mathematics, if taught formally, without the establishment of appropriate problem situations, without interesting content that reflects their application in real life, as well as without the active participation of all students, can quickly become boring, which will certainly affect the quality their achievements (Egerić, 2008; Eccles, 1999; Kathleen, 1996; Ćebić, 2000).

In the teaching of mathematics, it is necessary for students to become active participants in the teaching process who independently, with some help from the teacher, adopt mathematical content. According to Prvanović (1970: 83), the organization of mathematics teaching, as well as the entire work in mathematics education, takes place in the sign of independent formation of mathematical concepts, independent problem solving, independent finding of rules, discovery of facts and laws in the sign of permanent creative work of students. In line with this statement, some authors (Azuka, 2013; Dumma, 2009) consider learning based on the experiences of students and teachers to be an important determinant in active learning of mathematics, which they call experiential learning. These authors also believe that the mathematics teacher should abandon the verbal way of teaching the curriculum and accept a method based on student activities. In such forms of learning, according to the mentioned authors, it is necessary for teachers to understand their students and to understand their needs, as well as the problems they encounter in mastering mathematical requirements. Such an approach will ensure better cooperation between teachers and students, students will be motivated to learn mathematical content, which will be reflected in achieving a higher level of success in teaching mathematics. One of the basic tasks set before mathematics teachers is the creation of such teaching situations in which students will adopt teaching contents, form mathematical concepts and develop mathematical thinking through their own activity. The role of the teacher in this process is not reduced to "teaching" mathematical content, but rather implies guiding or helping students to learn mathematical content and solve tasks and problems in this subject.

For successful organization and quality realization of mathematics teaching, it is necessary to respect the stated specifics, which requires teachers to fully realize their professional role in all areas of teaching. The mathematics teacher should know mathematical contents to a greater extent than provided by the program, to carefully plan and prepare mathematics lessons, to know all his students well (intellectual abilities of students and the emotional side of their personality), as well as possible problems with

which the student encounter this not only in mastering mathematical content, but in general. Therefore, the teacher should adapt the teaching of mathematics to the needs and interests of students, to achieve quality communication and interaction with students, to enable each student to feel satisfaction in progress in learning mathematical content, as well as to students develops a love, not a fear of mathematics. Also, the teacher should be motivated, creative and persistent in his work, because without these characteristics of the teacher it is almost impossible to ensure quality teaching of mathematics, as well as a qualitatively higher level of student success in this subject.

The role of teachers of mathematics in the field of organization, planning, preparation and implementation of teaching

It is generally accepted that a quality organization of any activity enables its successful realization and achievement of efficient results, with minimal consumption of power, energy, time and other important resources. Before a deeper consideration of the issue and efficiency of the role of teachers in the organization of mathematics teaching, it is necessary to explain more precisely the meaning of the term "organization". This term is complex, ambiguous and used for various purposes. One of the most general definitions of the term "organization" implies the merging of individual parts into one whole, so that this whole is capable of life (Vujaklija, 1980: 639). This definition is close to the biological and mechanistic understanding, where the key determinant is the merging of certain organs (parts) into one whole, organism or apparatus. In a closer definition of the term "organization" it is necessary to pay attention to the terms "organizer" and "organizing".

From a sociological or social point of view, these terms contain an "editorial" (creative) character and refer to someone who according to a set plan edits, creates, arranges, shapes and formats a certain environment, group, association and the like, in order to achieve certain goals. Having in mind the presented concepts, in this context we can talk about the organization as a specific, planned social process in which individuals, as members of the group, perform their intended roles. Based on Luman's understanding (Luhmann, 1994: 190, according to: Pusić, 2005: 147), the organization implies a social structure in which a certain number of people establish mutual relations, in order to achieve the set goals by different methods and using different means. In the presented understanding, the importance of using different methods and tools that people use to achieve the planned pipes is noticed, and Luman attaches special importance to communication as a basic means of communication between people. Sikavica and Novak (1999: 33) view the "organization" in a similar way, according to which it represents a conscious association of people whose goal

is to fulfil certain tasks by appropriate means, with the least possible effort in any area of social life. Insight into the above concepts can be seen two-sided meaning of the term “organization”. On one hand, this term implies an integrated whole of different elements, in which each of them has a certain meaning, while on the other hand it represents the process in which certain purposeful activities are performed. It should be noted that when conceptually defining an organization, it is necessary to equally respect the stated meanings, because only such an approach can understand all its characteristics and specifics.

When it comes to teaching as a process aimed at acquiring knowledge and developing abilities and skills of students, the term “organization” in the broadest sense means designing, preparing, planning, implementation, or overall design, modeling and structuring of all elements in a certain, systematized way, which would enable the achievement of imagined or set goals. The organization of the teaching process takes place on several levels in which many different factors participate. In this regard, there are different understandings of the organization of teaching, so we can talk about the macro-organization and micro-organization of teaching, the organization of teaching in the form of curricula for different educational levels, the organization of teaching by type of school and the like. From the point of view of this paper, Stevanović’s understanding of the organization of teaching can be presented as one of the relevant determinations. Namely, Stevanović (1998) believes that the organization of teaching implies the activity of teachers that takes place through three phases of teaching, namely: planning of teaching work, preparation for teaching and teaching. However, having in mind the fact that students also actively participate in the teaching process, the organization of teaching should be viewed in a broader context, so that it represents a joint activity of teachers and students (participants) in planning, preparing and implementing the teaching process.

Teaching activity is a complex process that takes place according to certain laws, has its own course, movement and duration, so the greatest responsibility within its design and implementation belongs to the teacher. Based on this, Vilotijević (2001) believes that only a teacher who has a thorough knowledge of all the structural elements of the teaching process, as well as their mutual functional relations and connections can successfully plan and conduct classes.

When it comes to teaching in general, as well as teaching mathematics, quality organization provides systematic work, identifies and eliminates certain shortcomings, increases student productivity in achieving better results, which also gives the teacher additional strength to work with students in school. Whether it is

teachers who are at the beginning of their teaching career or teachers with longer work experience in teaching, a good organization of work in teaching mathematics in it largely determines the quality of teaching, the end result of which is student success. Also, quality organization of mathematics teaching, adequate planning, preparation and implementation of teaching activities are important from the point of view of developing good communicative and interactive relations between teachers and students, as well as from the point of view of students’ active participation in teaching and developing their interests in learning mathematical content.

Planning, preparation and implementation of mathematics

When it comes to teaching planning, the pedagogical, i.e. didactic and methodological literature most often mentions macro-level planning (planning in relation to the entire mathematics education, as well as teaching planning for the whole school year) and micro-level teaching planning. teaching for each lesson or teaching topic). From the point of view of this paper, the problem of planning mathematics teaching is considered in the context of daily teaching planning and includes all activities of teachers (and students) that are performed before, during and after the implementation of the lesson.

As shown in the earlier part of this paper, well-organized teaching of mathematics is one of the basic conditions for the successful realization of teaching goals and tasks. However, the organization of teaching is a complex process in which teachers are faced with different requirements that they need to meet. Well-organized teaching, for example, implies active participation of students, a high degree of motivation, quality communication and interaction between teachers and students, working atmosphere, constant evaluation of the organization and effects of teaching, application of different teaching methods, individualization of teaching process, use of modern educational technology. and similar (Špijunović and Maričić, 2016: 150). All of the above should be taken into account by the teacher already in the planning and preparation phase of teaching. Also, when preparing for a lesson, the teacher should think about the students, about their intellectual abilities, as well as about their ability to master mathematical content. When it comes to mathematical content, the fact is that they are logically related, but differ in their complexity and difficulty. Some of them are more complex and difficult than others, so students need more cognitive effort to understand them (Kurnik, 2009). In accordance with the above, many authors believe that during the organization of mathematics teaching, the focus of their activities and work should be focused on the preparation and planning of teaching and to devote most of their attention and available time to this teaching area (Ivić, Pešikan and Antić, 2001; Špijunović and Maričić, 2016; Schunk., et al. 2008).

The extent to which mathematics teaching will be well realized largely depends on the preparation and planning, as well as on the methodological design of each individual lesson, which implies specific procedures of mathematics teachers in choosing teaching methods, forms of teaching work, teaching aids, as well as from their mutual integration and combination. In order for the mathematics teacher to prepare and methodically shape the teaching in the best way, it is necessary to harmonize the total corpus of teaching activities with the didactic-methodical bases of mathematics teaching, such as teaching principles, types of classes and others.

Planning and preparation of mathematics classes in accordance with the principles of teaching

The basic guidelines and ideas on the basis of which the teaching, preparation and performance of mathematics are planned are contained in the teaching (didactic/methodological) principles. Teaching principles are determined in accordance with the needs of social development, according to the goals and tasks of teaching, as well as according to the characteristics of students and their ability to acquire knowledge and develop adequate skills in teaching. In addition to the theoretical, didactic principles have a practical significance, which is reflected in helping teachers to make appropriate decisions about procedures during the planning and implementation of teaching. However, regardless of their importance in teaching, there is still no clearly defined number of teaching principles in the pedagogical literature [3]. The reasons for this are numerous and can be traced to the non-uniformity of teaching practice; in the developmental characteristics of the teaching process; different understandings of didactics and methodologists on the relationship between the legality of teaching, teaching principles, teaching rules and the like. The impossibility of establishing the exact number of teaching principles is conditioned by the fact that some of them become obsolete, and in their place come new, current principles of teaching. Also, often the realization of one didactic principle fulfils the requirements of others, which indicates their complexity, mutual conditioning and intertwining. Without intending to analyze the problems of classification and the number of teaching principles, from the point of view of this paper it is necessary to pay attention to the teaching principles that most often occur in all previous classifications, and which are of great importance in planning and implementing mathematics teaching. Some of them are as follows: (1) the principle of scientific; (2) the principle of moderation of teaching according to the age and abilities of the student; (3) the principle of individualization; (4) the principle of systematicity and gradualness; (5) the principle of obvious teaching and (6) the principle of permanence of knowledge, skills and habits.

According to the principle of science, the process of organizing mathematics teaching should be based on scientific knowledge, which means, among other things, that the teacher should take into account not only the scientific interpretation of teaching content, but also its system-conceptual, organizational and operational components (Spijunovic and Maricic, 2016: 96). In the conditions of constant development of mathematics as a science, the number of information, data and mathematical concepts increases, which significantly affects the possibilities of their adequate presentation in teaching. The effectiveness of the teaching approach is reflected in the correct choice of scientific knowledge and finding ways for students to adopt it. Respecting the principle of science, one of the most important tasks of teaching mathematics is achieved, and that is the development of a scientific view of the world, as well as the development of mathematical (critical) thinking. Of course, as part of training students to develop critical thinking and a scientific view of the world, care should be taken to adapt mathematics teaching to their abilities and capabilities. This involves, for example, presenting mathematical concepts and content in ways that will ensure that all students have a full understanding and adoption, not just a few. Almost all mathematical contents, regardless of their abstractness, have certain characteristics that are understandable to all students, and it is up to the teacher to find them and explain to students in appropriate ways their essence or their relationship to the scientific concept to be adopted. The principle of science implies, also, the need to connect mathematical contents, i.e. the need to learn new mathematical contents on previously acquired and developed mathematical knowledge, which would serve as a basis for the adoption of more demanding mathematical contents. So, in addition to the requirement that mathematics teaching be based on scientific knowledge, teachers are expected to be patient in working with students and not to move on to processing new content if students have not met the previous requirements. The presented expectations from teachers speak in favor of the fact that teaching principles are intertwined and that it is almost impossible to separate them, i.e. to ignore some of them. Students can be trained for mathematical thinking and scientific (mathematical) view of the world, which is a basic requirement of the principle of science, provided that the mathematics teacher already in the process of planning and preparing for teaching, as well as during the implementation of teaching equally respects all teaching principles. It guides students through the teaching process.

In addition to the principle of science, in the teaching of mathematics it is necessary to adhere to the principle of moderation of teaching according to the age possibilities and abilities of students. By accepting this didactic principle, opportunities are created for

the teaching of mathematics to be in the function of optimal engagement and intensive development of the personality of each student (Špijunović, 2007). The principle of moderation is closely related to determining the degree of difficulty of mathematical tasks. Namely, the respect of this teaching principle excludes the possibility that students are given too easy or too difficult mathematical tasks. Too easy tasks can give students the wrong impression that mathematics is easy to learn or even uninteresting, which can reduce their interest and motivation to learn mathematical content, as well as reduce their level of active participation in the overall process of learning mathematics. This approach can especially hurt better students, because too easy tasks do not cause them to invest cognitive effort, do not encourage them to think, which in turn causes a decrease in concentration, lack of interest and boredom.

Frequent and long-term assignment of such tasks can cause students to give up learning and achieve poorer results in their achievement. When it comes to too difficult tasks, the situation is similar for all students, especially for those who have problems in acquiring teaching content and solving tasks in teaching mathematics. Too difficult tasks and requirements in mathematics teaching give students the impression that they are not able to master the given tasks, which can cause various consequences. The most common consequences are less successful, however, they can be more significant, so that they can cause students some form of anxiety (Schwartz, 2000) or fear of mathematics, which can result in their complete absence from participation in mathematics teaching, as well as absence learning other teaching contents and subjects (Frenzel, *et al.* 2007). Mathematics is a difficult subject, however, in teaching appropriate to students and their abilities, it can be learned and mastered with the investment of an optimal level of cognitive effort (Kurnik, 2009). Of course, the cognitive effort mentioned by Kurnik should not be understood as straining and straining the mind, but as an important factor in achieving and developing students' ability to learn mathematical tasks and content. The principle of moderation of teaching implies such planning of teaching requirements that will for a certain degree be above the current abilities of students. This idea, with which Vygotsky greatly contributed to the development of pedagogical psychology, as well as didactics, should be guided by mathematics teachers, already during the planning and preparation of classes.

Insight into the characteristics of the principle of moderation of teaching according to the possibilities of students can show the interconnectedness of all teaching principles, which was discussed in the earlier part of the text, as well as the importance of each of them individually. Thus, for example, the previous teaching principle is followed by the principle of individualization, the

respect of which is necessary for the implementation of quality teaching, when it comes to mathematics education. The fact is that in one class (class) there are students of approximately the same age. However, they differ significantly from each other. Each of the students has his own characteristics and traits. Thus, for example, some students achieve better learning success while others are weaker, some come from families in which better learning conditions are provided compared to others. In addition, students differ in character traits, as well as temperament, some are more extroverted and active in teaching, their activities and issues require the attention of teachers, while others are introverted and withdrawn, and in situations where they possess certain knowledge avoid avoiding. Also, some students are able to quickly acquire certain knowledge in the field of mathematics, while others need more time, some students find teaching mathematics interesting and interesting, while others have developed affinities for other subjects or extracurricular activities.

Differences in the individual characteristics of students require teachers to fully respect the principle of individualization in teaching when planning teaching work and to determine the types and difficulty of teaching tasks according to each individual student. This is by no means an easy task and requires teachers to invest a lot of time, effort and perseverance in the organization and implementation of the teaching process. It is necessary to adjust the teaching tasks and problems to enable each individual student and each of them to feel satisfied when solving mathematical problems. In addition, this principle implies planning a teacher's approach to each student, because the teacher is not only the organizer and manager of the teaching process, but also a counselor, associate, one who helps, who understands students and their problems. In situations when teachers meet the mentioned requirements, students become more active, independent, with a higher degree of self-confidence and more ready to take the initiative in their work. In order to achieve this, it is necessary to achieve a certain degree of freedom in mathematics teaching in terms of encouraging active learning, enabling students to participate in the selection of teaching content and tasks and achieving good communication and quality interaction in teaching. mutual respect and cooperation of all students in the class.

During the planning and implementation of teaching work based on respect for the importance of the previous three principles, teachers should not lose sight of the importance of the principles of gradual and systematic teaching. The significance of this teaching principle is reflected in the understanding that the teaching process of mathematics is understood as a process of superstructure (Schwartz, 2000), in which each subsequent step in

learning is conditioned by mastering the previous one. The idea of gradualness and systematicity in teaching was first pointed out by Comenius, and later developed by Herbart, Disterweg, Usinski and others. In this didactic principle, there are two requirements that need to be met (gradual and systematic), which most often occur in a single form. Graduality in teaching implies the adoption of first elementary, basic content, and then more complex and demanding. When it comes to systematicity, it implies the inductive adoption of mathematical content, systematically and gradually, taking into account the level and quality of previously acquired knowledge. The principle of gradualness and systematicity is present in the entire process of teaching mathematics, requiring that all teaching contents, teaching methods, forms of work, as well as the use of teaching aids be logically connected and systematized into an appropriate unit, adapted to teaching conditions.

In addition to the above, the principle of obviousness occupies a significant place already during the planning, as well as in the realization of mathematics teaching. This teaching principle is based on sensory perception, the importance of which was first pointed out by Aristotle, and later by Comenius, Piaget, and others. The basic meaning of the principle of obviousness in mathematics teaching is reflected in the fact that abstract mathematical concepts are brought closer to students and thus facilitate their understanding and adoption. Respect for the principle of obviousness is especially important for younger students. In the early age period, students most easily acquire mathematical concepts with the help of concrete examples, as well as with the use of adequate teaching aids and materials. However, when it comes to older students of primary school, the principle of obviousness should not be understood only in terms of the organization of teaching based on the immediate senses, but its importance should be viewed in a broader context. Teaching based on sensory knowledge certainly has its advantages, when it is possible to use adequate teaching aids, pictures, objects, films and the like. In situations where this is not possible or when it comes to concepts that are practically impossible to adopt with concrete examples, the participation of the principle of obviousness implies the organization of teaching based on previous experiences of students. In accordance with the above, Bakovljević (1966: 463) points out that teaching can be obvious even without obvious teaching aids, while on the other hand it does not have to be obvious even with the help of a large number of such means. Thus, the quality of teaching depends on whether the process of acquiring knowledge is based on previously acquired experience of students, regardless of whether their experience is gained through direct participation in certain situations or through the teacher's explanations.

It should be noted that the principle of obviousness cannot and must not be an end in itself, because teaching conceived in this way would lose its meaning. Teaching mathematics should develop students' knowledge and skills that will have use value. This means that based on the acquired knowledge, students will be able to solve various tasks and problems, not only in the teaching of mathematics but also beyond. At the same time, the acquired knowledge and abilities of students in mathematics should be a good basis for acquiring new knowledge, as well as developing critical or mathematical thinking. Without teaching based on the principle of obviousness in its broadest sense, these tasks are difficult to accomplish.

Respecting the presented teaching principles within the preparation and implementation of mathematics teaching enables the realization of other important requirements. These are requirements contained in the principle of permanence of acquired knowledge, skills and habits. The essence of this teaching principle is that the acquired knowledge and ability of students to solve mathematical problems should become their permanent property or even features, so that the acquired knowledge can be used and applied by students in different teaching and life situations. The permanence of students' knowledge, skills and habits in teaching mathematics are conditioned by the ways in which they are acquired and determined. If the process of learning in mathematics teaching is based on the principle of science, if the teaching contents are systematically and gradually classified according to individual abilities of students, if students actively participated in that process, if they understood teaching material and if teaching is continuously based on continuous connection of previous teaching content with new, then certainly the acquired knowledge of students, as well as their ability to solve mathematical problems and problems will certainly be more permanent.

Based on the presented principles of teaching, it can be concluded that the planning and preparation of mathematics teaching is one of the most important areas of teaching. Mathematics teachers are faced with a large number of requirements, the implementation of which is by no means simple. In order to be efficient in teaching, i.e. to improve the quality of teaching and contribute to achieving a higher level of student success, teachers should plan classes and implement planned activities in accordance with the above teaching principles. In doing so, account should be taken of their intertwining, i.e. conditionality, and it is not recommended to neglect any of the presented didactic principles. Also, it should be emphasized that the presented classification of teaching principles does not represent their hierarchy, because the hierarchical classifica-

tion, when it comes to teaching principles, is neither possible nor necessary to establish. Each teaching principle has its own meaning and none of them can be subordinated or superior to another. Also, it should be emphasized that their importance needs to be seen in a broader context, as well as that it is necessary to take into account the didactic principles that are not elaborated in this brief.

Thus, it can be concluded that well-planned and prepared teaching of mathematics largely depends on the fact that teachers adhere to the requirements and recommendations presented in the didactic principles, and that it is necessary to observe the presented principles in their harmonious unity. Such an approach will enable a quality organization of mathematics teaching, in which students will achieve a higher level of success in learning and together with their teachers will be satisfied with the results achieved in mastering mathematical content.

Planning and preparation of mathematics teaching harmonized according to the types of classes

One of the conditions for achieving a quality teaching process in the field of mathematics is the planning of teaching in accordance with the types, i.e. types of classes. Classes are the basic form of organization of school work in which educational, upbringing and practical goals of teaching are achieved. There are a number of different criteria in the didactic literature, according to which the types of classes are determined, of which the goal of the class, teaching materials, teaching methods and teaching tasks are most often singled out (Vilotijević, 1999; Dašić, 2008; Kurnik, 2007). It is not uncommon for authors to view the typology of lessons, because such an orientation of teaching shapes and patterns the work of teachers, thus limiting their freedom and creativity in teaching (according to: Kurnik, 2007: 99). However, the need to classify classes is useful precisely because of the fact that teaching is a planned and organized process, and that too much freedom in designing and planning classes could cause more negative than positive effects. Some authors believe that the classification of classes is necessary precisely to allow multiple choices in the planning and organization of classes, which acts as a counterbalance to the effort to create a universal recipe, according to which each class would be organized, while others believe that the classification of classes increases opportunities for improving teaching work, as well as opportunities for the development of mathematics teaching methodology (Kurnik, 2007; Prodanović and Ničković, 1998; Vilotijević, 1999). According to the most commonly used classification, in the didactic and methodological literature, the following types of mathematics classes can be distinguished: (1) class of processing new teaching material; (2) the time of determining the teaching material and (3) the time of checking the learned material.

The processing time of new teaching material in mathematics is important because the overall quality of teaching, as well as the success of students, largely depends on the way it is organized and implemented. In this class, the most attention and time is devoted to introducing students to new mathematical concepts, contents, rules and the like (Dašić, 2008; Kadum, 2007; Špijunović and Maričić, 2016). In order to achieve the goals and tasks of teaching mathematics, in the classes of processing new teaching materials, it is necessary to carefully choose the teaching content, determine the optimal scope and depth of their processing, choose methods, forms and means of teaching that will make it easier for students adoption of teaching contents and achieving better learning outcomes. The class of processing new teaching material is not isolated from other classes, which can be seen in its structure. Namely, it consists of several different stages, which, in addition to processing new content, also include the stage of repeating previous content, as well as the phase of determining and checking what has been learned. The repetition of previous contents represents the introductory phase in the processing of new teaching material, in which the psychological preparation of students for work is established and their thinking is activated, as well as participation in teaching (Kadum, 2008; Kurnik, 2007). On the other hand, the determination phase involves checking the degree of adoption of the teaching material, in terms of obtaining feedback from students on the success of the implementation of the lesson.

When it comes to the lessons of determining the teaching material in the teaching of mathematics, their basic function is contained in one of the teaching principles, namely the principle of permanence of acquired knowledge, skills and habits. Determining the teaching material is usually achieved in two ways, by repetition and practice. It should be emphasized that these classes are not a typical repetition of teaching materials, which would reduce the activities of students to the reproduction of previously learned and memorized teaching content and mathematical tasks. The essence of repetition is that on the basis of previously learned mathematical contents, new teaching situations and tasks are created in which, by noticing certain connections and relationships between them, students will look for ways to solve them. Thus conceived lessons of repetition of teaching material, in addition to developing new approaches in solving mathematical tasks, encourage the degree of motivation to learn mathematical content and enable better development of critical (mathematical) thinking (Kieren, 2004; Schwartz, 2000). Therefore, some authors (Špijunović and Maričić, 2016; Wigfield and Eccles, 1992) recommend that during the determination of mathematics teaching material, the focus of teaching should be on the individual abilities and interests of students, taking into account that the number of completed tasks per hour

is not so important, how important it is to point out the existence of different ways in which it is possible to solve even one and the same task. Repetition and practice in mathematics classes play a significant role, so according to some classifications there are special types of classes in which these two activities are predominantly represented (Prodanović and Ničković, 1988; Špijunović and Maričić, 2016). However, even in such classes the emphasis is placed on wider complexes of teaching contents, so that they are most often organized, for example, after mastering several teaching units within one teaching topic.

The lesson of checking the learned material in mathematics teaching also occupies a significant place. Although the testing of students' knowledge is carried out continuously in each lesson of mathematics, there is a justified need for the organization of special classes, in which the acquired knowledge of students will be more fully and deeply tested. Namely, one of the goals of checking the acquired knowledge of students in mathematics teaching is to determine the quality of organization, i.e. realization of teaching, on the basis of which further planning and design of joint work and activities of teachers and students is carried out. In addition, the knowledge test enables the ranking of students according to the degree of success in teaching mathematics. This ranking is also necessary in the further planning and implementation of teaching because it helps the organization of differentiated teaching, individualized teaching, as well as the selection of teaching methods and forms of teaching, all aimed at achieving a qualitatively higher level of student success in mathematics.

Insight into the presented typology of classes, in addition to the basic characteristics on the basis of which the mentioned classes differ, their interconnection can be noticed, but also the complexity of the teaching role. It is noticeable that all classes, in addition to the dominant didactic functions presented in their names, contain certain elements of other classes. Thus, for example, in the classes of processing new teaching materials, activities such as repeating previous teaching contents, i.e. determining teaching materials, occupy a significant place. Thus, no class is based on only one teaching activity, regardless of whether it is about determining the previous or processing new teaching material. Therefore, all classes can be considered to be combined. Although the didactic and methodological literature does not mention the combined class as a special type of class, it is understood that there are no completely "pure" classes, based on only one didactic function. It is necessary to use the elements of all presented classes in each class (as many different conditions allow) because in such circumstances not only the activity of students increases, but also the activity of teachers, as well as the quality of teaching. With such an approach in the

work, teachers encourage students to think, to learn earlier content, to present their ways and procedures in solving mathematical problems, to discuss certain problems, to present their ideas and the like. At the same time, teachers have an insight into students' activities, how far they have progressed in learning, what difficulties students encounter while learning mathematical content and more, which certainly contributes to better teaching of mathematics and achieving better student results. In addition to the interconnectedness and conditionality of classes, the presented typology also shows the requirements of the previously discussed didactic principles, which also speaks in favor of the complexity of teaching. In accordance with the requirements of didactic principles, didactic functions of classes, characteristics and abilities of students, teachers should further plan, methodically design and implement their classes in mathematics teaching [4-21].

Conclusion

In this paper, it is pointed out that the role of teachers, in the context of achieving quality teaching and a higher level of student success, is a significant area of scientific research. However, when it comes to research on the role of mathematics teachers, in the older grades of primary school, the fact is that there is significantly less research work. The accelerated development of science, technology and engineering has influenced the attention of scientists and experts in various fields to the importance of mathematics education in developed countries. Consequently, a large number of scientific papers have been observed in foreign literature in which problems in learning mathematics are investigated in different ways. This research is often focused on specific, individual teacher actions, such as: teaching organization; student participation in planning teaching activities; ways of presenting teaching materials; encouraging motivation through different actions of teachers; difficulties in motivating certain students; student motivation by applying mathematics quizzes; communication in mathematics teaching; mathematical anxiety; group form of teaching work in mathematics; discursive teaching in mathematics; connection of competitions in mathematics with the development of quality interactive relationships; use of computers and the Internet in teaching mathematics and the like. The presented examples indicate the need for a deeper study of problems in learning mathematics. In our country, for example, there is still no significant research on problems in the organization and implementation of mathematics teaching in the older grades of primary school. There are several papers in which the role of teachers, their importance in teaching, as well as problems in teaching are explored in a broader context, without a more specific focus on teaching mathematics, or problems in teaching mathematics to older students in primary school. Having in mind the obtained research results, as well as the importance of learning

mathematical contents, there is a need for more active, continuous and complete research of problems in the teaching of mathematics among older school students.

When it comes to teachers whose students achieve a lower level of success in this subject, it is necessary to emphasize the following: for example, it can be noticed that these teachers still pay significant attention to the presentation of teaching materials, in the sense that the teacher explains while students listen and they write. Also, these teachers have different motivational procedures and quality communicative relations between teachers and students, but not to the extent that is the case with teachers whose students achieve a higher level of achievement in mathematics. In this regard, it should be emphasized that the previously presented recommendations can help these teachers to significantly improve the quality of their teaching work and cooperation with students, which will certainly be reflected in a higher level of student achievement. Teachers need to understand the essence and importance of social and educational changes and to direct their activities in a direction that will significantly increase student participation in the teaching process. Teachers are not only expected to adapt to changes in education, but to become the bearers of these changes, which means better organization of teaching work, a higher degree of creativity in teaching and a better interactive relationship with students.

It can be concluded that most of their procedures in mathematics teaching are directed by teachers towards external motivational incentives, while activities aimed at developing students' interests are realized by a smaller number of teachers. Therefore, attention should be paid to the results presented in this research, and it can be seen that a higher level of success in mathematics is achieved by those students whose teachers, in addition to external incentives, often organize activities that stimulate students' interest in mathematics.

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