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Research Article

Determining the Flexibility between Asymmetric Models

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Abstract

Adjusting shape parameter in a model makes the model more skewed. Therefore, introducing a shape parameter to any model also makes it more flexible to fit/handle non-normal data. In this paper, a univariate model called Lehmann Type II modified weighted Rayleigh model is developed from modified weighted Rayleigh distribution; likewise, Lehmann Type II modified weighted Exponential model is developed from modified weighted Exponential distribution through beta generalized link function. Performances of both models are compared with their baseline distributions. Some basic statistical properties of the new models including; moments, generating function, survival function, hazard function is derived. Parameter estimates are obtained via method of maximum likelihood estimation. Model selection criteria were employed to select the better among the models. An application to a real data set is given to show the flexibility and performance of the distributions.

Keywords: Beta Link Function; Lehmann Type II; Shape Parameter; Weighted Exponential; Weighted Rayleigh

Introduction

Weibull distribution is a continuous probability distribution. Versatile, most widely used distribution and has been a powerful probability distribution in reliability analysis; weighted distributions are used to adjust the probabilities of the events as observed and recorded. Rayleigh distribution is a special case of Weibull distribution which has been widely used for modeling lifetime data in a wide variety of areas including: survival analysis and engineering, while exponential distribution also is one of the most widely used distribution like Weibull distribution. This distribution is often used to model the time elapsed between events. Many works have been done extensively on each one of the distributions in literature either on univariate, bivariate or multivariate. At the same time, various studies have been done on convolution of two or more distributions to derive a new distribution. For instance, modified weighted Weibull distribution by Aleem., *et al.* [1], in their paper they obtained sub-distributions e.g. modified Weighted Exponential, modified weighted Rayleigh and modified weighted extreme value distribution. Shahbaz., *et al.* [2] studied on a class of weighted Weibull distribution and its properties. In literature, Lehmann type II distribution is very few and scanty. Meanwhile, Lehmann type II is a special case of and beta distribution in conjunction with other distribution(s) using beta link function by Jones [3]. This includes; Badmus., *et al.* [4] worked on Lehmann type II weighted Weibull distribution, the proposed distribution was obtained from beta weighted Weibull distribution by letting one of the additional shape parameters to be one (when a=1). Amusan and Khalid [5] investigated on a comparative analysis on the performance of Lehmann type II inverse Gaussian model and standard inverse Gaussian model in terms of flexibility. In this study we extend the modified weighted Rayleigh and modified weighted exponential distribution by adding a shape parameter to the existing distributions as earlier mentioned.

The paper is organized as follows: section 2 consist the density function (pdf), distribution function (cdf), reliability, hazard rate functions of the proposed distributions. Section 3 contains the derivation of the moments and moment generating function, skewness, kurtosis and entropy. Estimation of model parameters using method of maximum likelihood estimation is presented in section 4. Section 5 contains application of the proposed models to nicotine measurements data in order to compare their performance and flexibility. The discussion and result were discussed in Section 6 and concluded in section 7.

Material and Method

Lehmann Type II Modified Weighted Rayleigh (LMWR) and Lehmann Type II Modified Weighted Exponential (LMWE) Distribution.

Here we introduced a shape parameter into the existing Modified Weighted Rayleigh (MWR) and Modified Weighted Exponential (MWE) distribution by Aleem., *et al.* [1] to generate more skewed distribution called LMWR and LMWE distribution. The pdf of the existing MWR and MWE; and their corresponding cdf are given as

$$f_{MWR}(x) = 2\alpha(\beta\gamma^2+1)\, x e^{((-\alpha(\beta\gamma^2+1)\, x^2)} \qquad (1)$$

$$F_{MWR}(x) = 1 - e^{((-\alpha(\beta\gamma^2+1)\, x^2)}$$

and

$$f_{MWE}(x) = \alpha(\beta\gamma+1)\, e^{((-\alpha(\beta\gamma+1)x)} \qquad (2)$$

$$F_{MWE}(x) = 1 - e^{((-\alpha(\beta\gamma+1)x)}$$

where, α is a scale parameter while, β and γ are shape parameters.

Meanwhile, we employ the beta link function by Jones [3] and letting one of the shape parameters equal to 1 (one); say a = 1 and this led us to the derivation of LMWR and LMWE as shown below: the Beta link function is given as:

$$f(x) = \frac{f(x)[F(x)]^{a-1}\,[1-F(x)]^{b-1}}{B(a,b)}, \; a,b > 0 \; (3)$$

where,

x >0, f(x)=d/dx F(x) and a>0,b>0; a shape parameter each is added to the existing (MWR) and (MWE) distribution, B(a,b)=Γ(a)Γ(b)/Γ(a+b) is the beta function, f(x),F(x) are pdf and cdf (MWR) and (MWE) respectively.

Then, if we let a = 1, beta link function in (3) becomes

$f(x)=b[f(x)[1-F(x)]^{b-1}]$, b > 0 (4)

Now, by substituting expressions in (1) and (2) into (4); to obtain the density function of LMWR and LMWE distribution, we have

$$f_{LMWR|(\alpha,\beta,\gamma,b)}(x)=b[e^{(-\alpha(\beta\gamma^2+1)}\, x^2]^{b-1}\, 2\alpha(\beta\gamma^2+1)\, x e^{(-\alpha(\beta\gamma^2+1)\, x^2)} \qquad (5)$$

and

$$f_{LMWE|(\alpha,\beta,\gamma,b)}(x)= b[e^{(-\alpha(\beta\gamma+1)x}]^{v-1}\, \alpha(\beta\gamma+1)\, e^{-\alpha(\beta\gamma+1)x} \qquad (6)$$

Equations (5) and (6) become the pdf of LMWR and LMWE distribution; and b is the shape parameter in addition to the exiting parameters in the baseline distribution.

The plots below are the pdf plots of both LMWR and LMWE distributions at different values of b= (5,3.5,2.5,1.5) and (1.5,1.3,1.2,1.1) when c=α,d=β and e=γ are fixed at (0.3,5,0.3) althrough. The values are initial values given to each parameter in order to achieve different shapes from each distribution and to detemine the distribution that could accomodates skewed data. Also, as the values of b decrease, the skewness of both LMWR and LMWE desrease and the graph skewed to the right as shown in figure 1.

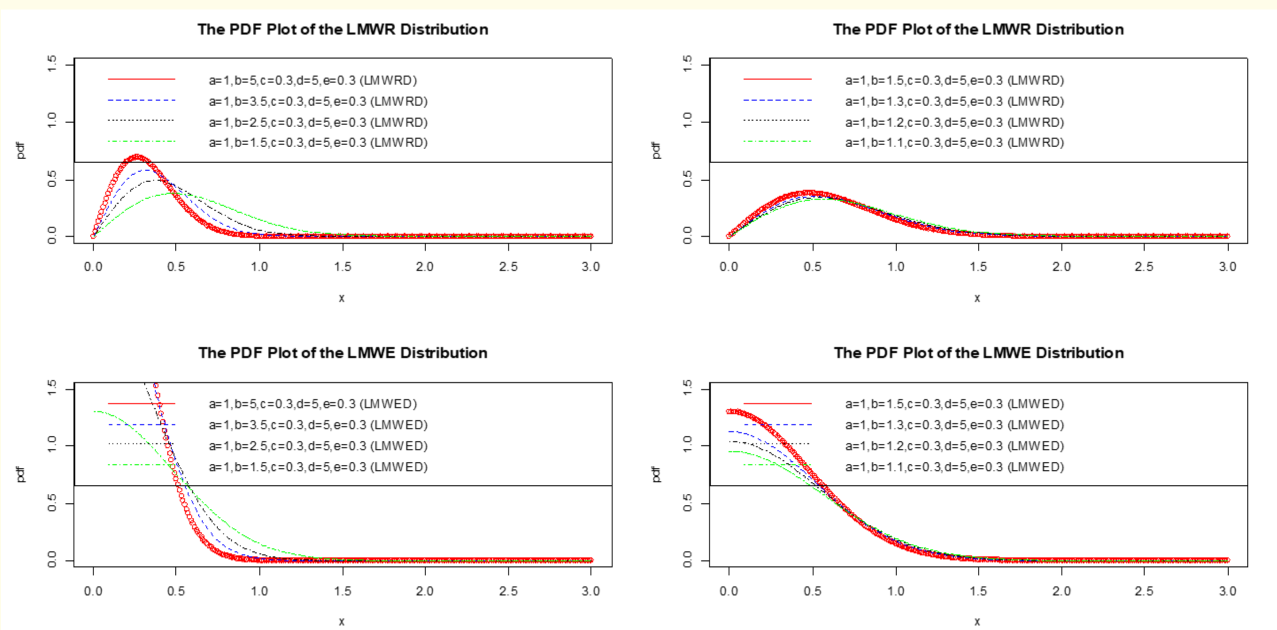

Figure 1: The PDF plots of the LMWR and LMWE distributions.

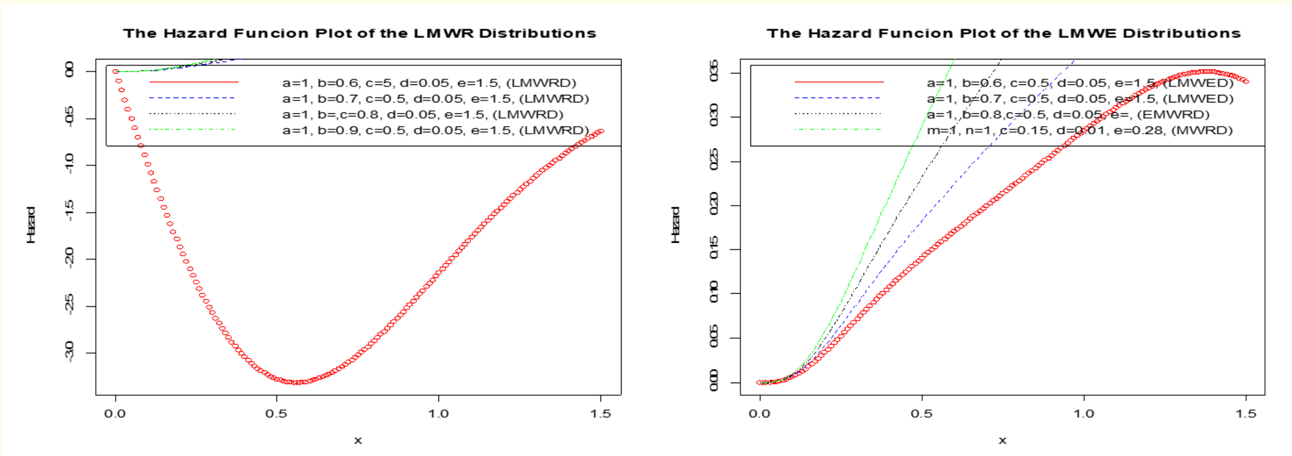

Figure 3: The plot of hazard function of the, LMWR= (b,α,β,γ) and LMWE = (b,α,β,γ) *((0.6, 5, 0.05, 1.5).*

$$M(x) = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_{-\infty}^{\infty} e^{tx} f_{MWR}(x)[F_{MWR}(x)]^{(i+1)-1}\,dx \quad (18)$$

Substituting the pdf $f_{MWR}(x)$ and *cdf* $F_{MWR}(x)$ as defined in (1) above into MGF M(x) in (18) gives

$$M_{LMWR}(x) = b\sum_{i=0}^{\infty}(-1)^i\binom{n-1}{i}\int_{-\infty}^{\infty} e^{tx}\left[e^{(-\alpha(\beta\gamma^2+1)x^2}\right]^{(i+1)-1} \left[2\alpha(\beta\gamma^2+1)xe^{(-\alpha(\beta\gamma^2+1)x^2}\right]dx \quad (19)$$

and Substituting the pdf $f_{MWR}(x)$ and cdf $F_{MWR}(x)$ as defined in (2) above into MGF M(x) in (18) yields

$$M_{LMWE}(x) = b\sum_{i=0}^{\infty}(-1)^i\binom{n-1}{i}\int_{-\infty}^{\infty} e^{tx}\left[e^{(-\alpha(\beta\gamma+1)x}\right]^{(i+1)-1} \left[2\alpha(\beta\gamma^2+1)e^{(-\alpha(\beta\gamma+1)x}\right]dx \quad (20)$$

Moments

According to Aleem., *et al.* [1], the r[th] non-central moment of the class of Modified Weighted Rayleigh distribution MWR(α,β,γ) is given as:

$$\mu'_{MWRr} = E(X^r) = 2(\beta\gamma^2+1)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\beta\gamma^2+1)^{-\frac{r}{2}-1} \quad (21)$$

The r[th] noncentral moment of the Lehmann Type II Modified Weighted Rayleigh distribution is given as

$$\mu'_{LMWR(r)} = \int_0^{\infty} x^r f_{LMWR}(x)dx$$

i.e.

$$\mu'_{LMWR(r)} = \int_0^{\infty} x^r \{b[1-U_1(x)]^{b-1}dU(x)\}$$

where

$$U_1(x) = e^{(-\alpha(\beta\gamma^2+1)g^2}, k_1(x) = e^{-\alpha x^2}, \tau_1 = (\beta\gamma^2+1)$$

Then,

$$\mu'_{LMWW(r)} = \left[b\left(2(\tau_1)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\tau_1)^{-\frac{r}{2}-1}\right)\right]\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}. \left\{\int_0^{\infty}[\mathrm{k}_1(\mathrm{x})(\tau)]^{(i+1)-1}dx\right\}$$

$$= \theta_1\left[2(\tau_1)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\tau_1)^{-\frac{r}{2}+1}\right] \quad (22)$$

where $\theta_1 = b\left[\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_0^{\infty}[\mathrm{k}_1(\mathrm{x})(\tau_1)]^{(i+1)-1}dx\right]$,

The first four non-central moments μ_r', by letting r=1,2,3 and 4 can be obtained respectively; in (22) i.e. μ_1' is given as

$$\mu_1' = E_{LMWR}(x) = \left[b\left(2(\tau_1)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\tau_1)^{-\frac{3}{2}}\right)\right]\left[\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\right]$$

Also, central moments μ_r, r=1,2,3,4,… are related to noncentral moments μ_r' as

Also, central moments μ_r, r=1,2,3,4,... are related to noncentral moments μ_r' as

$$\mu_r = \sum_{w=0}^{r} \binom{r}{w} \mu'_{r-w}\, \mu_w'^{w}, \text{ where } \mu_1' = \mu \text{ and } \mu_0' = 1 \ (23)$$

Conversely, the mean and variance, 3rd and 4th moments of the LMWR distribution are given by

$$\mu = \mu_1'$$
$$\mu_2 = \mu_2' - \mu^2$$
$$\mu_3 = \mu_3' - 3\mu\mu_2' + 2\mu^3 \text{ and}$$
$$\mu_4 = \mu_4' - 4\mu\mu_3' + 6\mu^2\mu_2' - 3\mu^4$$

where

$$\mu_1' = \theta_1\left[2(\tau_1)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\tau_1)^{-\frac{3}{2}}\right] \quad (24)$$

$$\mu_2' = \theta_1\left[2(\tau_1)\alpha^{-\frac{2}{2}}\Gamma\left(\frac{2+2}{2}\right)(\tau)^{-\frac{(2+2)}{2}}\right] = 2\theta_1[2(\tau_1)\alpha^{-1}(\tau_1)^{-2}] \ (25)$$

$$\mu_3' = \theta_1\left[2(\tau_1)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{3+2}{2}\right)(\tau_1)^{-\frac{(3+2)}{2}}\right] = 6\theta_1\left[2(\tau_1)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{5}{2}\right)(\tau_1)^{-\frac{5}{2}}\right] (26)$$

$$\mu_4' = \theta_1\left[2(\tau_1)\alpha^{-\frac{4}{2}}\Gamma\left(\frac{4+2}{2}\right)(\tau_1)^{-\frac{(4+2)}{2}}\right] = 24\theta_1[2(\tau_1)\beta^{-2}2(\tau_1)^{-3}] \ (27)$$

Moments measures of Skewness, $\omega_{SK(1)}$ and of excess kurtosis, $\omega_{KT(2)}$, of LMWR distribution are respectively given as

$$\omega_{SK(1)} = \frac{\mu_3}{\sqrt[2]{\mu_2^3}} \qquad (28)$$

$$\omega_{KT(1)} = \frac{\mu_4}{\mu_2^2} - 3 \qquad (29)$$ (see Badmus., *et al.* [6])

In this same vein, we followed the same way we obtained the first-four non-central moment of LMWR distribution also obtain for LMWE distribution as follows:

$$\mu'_{LMWE(r)} = \int_0^\infty x^r \{b[1 - U_2(x)]^{\nu-1} dx\}$$

where

$$U_2(x) = e^{(-\alpha(\beta\gamma+1)x}, k_2(x) = e^{-\alpha x}, \tau_2 = (\beta\gamma + 1)$$

Also,

$$\mu'_{LMWE(r)} = \left[b\left((\tau_2)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\tau_2)^{-\frac{r}{2}-1}\right)\right]\sum_{i=0}^{\infty}(-1)^i\binom{\nu-1}{i}.$$
$$\left\{\int_0^\infty [\mathrm{k}_2(\mathrm{x})(\tau_2)]^{u(i+1)-1} dx\right\}$$
$$= \theta_2\left[(\tau_2)\alpha^{-\frac{r}{2}}\Gamma\left(\frac{r}{2}+1\right)(\tau_2)^{-\frac{r}{2}+1}\right] \qquad (30)$$

Badmus., *et al.* [7]

where $\theta_2 = b\left(\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_0^\infty [\mathrm{k}_2(\mathrm{x})(\tau_2)]^{u(i+1)-1} dx\right)$,

We also derived the first four non-central moments μ_r', by letting r=1,2,3 and 4 respectively in (30); i.e. μ_1' is given as

$$\mu_1' = E_{LMWE}(x) = \left[b\left((\tau_2)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\tau_2)^{-\frac{3}{2}}\right)\right]\left[\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\right]$$

Hence, central moments μ_r, r=1,2,3,4,... are related to noncentral moments μ_r' as

$$\mu_r = \sum_{w=0}^{r}\binom{r}{k}\mu'_{r-k}\,\mu_k'^{k}, \text{ where } \mu_1' = \mu \text{ and } \mu_0' = 1 \ (31)$$

Then, the mean and variance, 3rd and 4th moments of the LMWE distribution are given by

$$\mu = \mu_1'$$
$$\mu_2 = \mu_2' - \mu^2$$
$$\mu_3 = \mu_3' - 3\mu\mu_2' + 2\mu^3 \text{ and}$$
$$\mu_4 = \mu_4' - 4\mu\mu_3' + 6\mu^2\mu_2' - 3\mu^4$$

where,

$$\mu_1' = \theta_2\left[(\tau_2)\alpha^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)(\tau_2)^{-\frac{3}{2}}\right] \quad (32)$$

$$\mu_2' = \theta_2\left[(\tau_2)\alpha^{-\frac{2}{2}}\Gamma\left(\frac{2+2}{2}\right)(\tau_2)^{-\frac{(2+2)}{2}}\right] = 2\theta_2[(\tau_2)\alpha^{-1}(\tau_2)^{-2}] \ (33)$$

$$\mu_3' = \theta_2\left[(\tau_2)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{3+2}{2}\right)(\tau_2)^{-\frac{(3+2)}{2}}\right] = 6\theta_2\left[(\tau_2)\alpha^{-\frac{3}{2}}\Gamma\left(\frac{5}{2}\right)(\tau_2)^{-\frac{5}{2}}\right] (34)$$

$$\mu_4' = \theta_2\left[(\tau_2)\alpha^{-\frac{4}{2}}\Gamma\left(\frac{4+2}{2}\right)(\tau_2)^{-\frac{(4+2)}{2}}\right] = 24\theta_2[(\tau_2)\alpha^{-2}2(\tau_2)^{-3}] \ (35)$$

Moments measures of Skewness, $\omega_{SK(2)}$ and of excess kurtosis, $\omega_{KT(2)}$, are respectively given as

$$\omega_{SK(2)} = \frac{\mu_3}{\sqrt[2]{\mu_2^3}} \qquad (36)$$

$$\omega_{KT(2)} = \frac{\mu_4}{\mu_2^2} - 3 \qquad (37)$$

Estimation of Parameter

We derived the maximum likelihood estimation (MLEs) of the parameter of the LMWR(α,β,γ,b) distribution, Cordeiro., *et al.* [8] and Badmus., *et al.* [7] by setting φ_1=(b,ρ,δ), where δ=(β,γ,θ) and is a vector of parameters. Then, the likelihood

$$L_{LMWR}(\varphi_1) = nlog\rho - nlogb + \sum_{i=1}^{n}\log[f(x;\varphi_1)] + (b-1)\sum_{i=1}^{n}\log[1 - F(x;\varphi_1)] \ (37)$$

$$L_{LMWR}(\varphi_1) = Const - nlogb + \sum_{i=1}^{n}\log[f(x;\varphi_1)] + (b-1)\sum_{i=1}^{n}\log[1 - F(x;\varphi_1)] \ (38)$$

Taking partial derivative of (38) with respect to (b,α,β,γ), we obtain

$$\frac{\partial L_{LMWR}(\varphi_1)}{\partial b} = -nlog(b) + (b-1)\sum_{x=1}^{n} \log\left[1 - F(x;\varphi_1)\right] \quad (39)$$

$$\frac{\partial L_{LMWR}(\varphi_1)}{\partial \alpha} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \alpha}[f(x;\varphi_1)]}{f[(x;\varphi_1)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \alpha}[1-F(x;\varphi_1)]}{1-F(x;\varphi_1)}\right] \quad (40)$$

$$\frac{\partial L_{LMWR}(\varphi_1)}{\partial \beta} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \beta}[f(x;\varphi_1)]}{f[(x;\varphi_1)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \beta}[1-F(x;\varphi_1)]}{1-F(x;\varphi_1)}\right] \quad (41)$$

$$\frac{\partial L_{LMWR}(\tau)}{\partial \gamma} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \gamma}[f(x;\varphi_1)]}{f[(x;\varphi_1)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \gamma}[1-F(x;\varphi_1)]}{1-F(x;\varphi_1)}\right] \quad (42)$$

(39) to (42) can be solved using iteration method (Newton Raphson i.e in order to obtain $\hat{b},\hat{\alpha},\hat{\beta},\hat{\gamma}$ the MLE of (b,α,β,γ) respectively.

Furthermore, the maximum likelihood estimation (MLEs) of the parameter of LMWE(α,β,γ,b) distribution following the same procedure:

Letting φ_2=(b,ρ,δ), where δ=(α,β,γ) and is a vector of parameters.

The likelihood is

$$L_{LMWE}(\varphi_2) = nlog\rho - nlogb + \sum_{i=1}^{n} \log[f(x;\varphi_2)] + (b-1)\sum_{i=1}^{n} \log\left[1 - F(x;\varphi_2)\right] \quad (43)$$

$$L_{LMWR}(\varphi_2) = Const - nlogb + \sum_{i=1}^{n} \log[f(x;\varphi_2)] + (b-1)\sum_{i=1}^{n} \log\left[1 - F(x;\varphi_2)\right] \quad (44)$$

Taking the partial derivative of (44) with respect to (b,α,β,γ), we get

$$\frac{\partial L_{BMWE}(\varphi)}{\partial v} = -nlogb + (b-1)\sum_{t=1}^{n} \log\left[1 - F(x;\varphi_2)\right] \quad (45)$$

$$\frac{\partial L_{LMWE}(\varphi_2)}{\partial \alpha} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \alpha}[f(x;\varphi_2)]}{f[(x;\varphi_2)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \alpha}[1-F(x;\varphi_2)]}{1-F(x;\varphi_2)}\right] \quad (46)$$

$$\frac{\partial L_{LMWE}(\varphi_2)}{\partial \beta} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \beta}[f(x;\varphi_2)]}{f[(x;\varphi_2)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \beta}[1-F(x;\varphi_2)]}{1-F(x;\varphi_2)}\right] \quad (47)$$

$$\frac{\partial L_{LMWE}(\varphi_2)}{\partial \gamma} = \sum_{x=1}^{n} log\left[\frac{\frac{\partial}{\partial \gamma}[f(x;\varphi_2)]}{f[(x;\varphi_2)]}\right] + (b-1)\sum_{x=1}^{n} \log\left[\frac{\frac{\partial}{\partial \gamma}[1-F(x;\varphi_2)]}{1-F(x;\varphi_2)}\right] \quad (48)$$

(45) to (48) can be solved using iteration method (Newton Raphson) to obtain $\hat{b},\hat{\alpha},\hat{\beta},\hat{\gamma}$ the MLE of (b,α,β,γ) respectively.

Application to real-data set

The two distributions were applied to 346 nicotine measurements data extracted from Handique and Chakraborty [9], to establish the supremacy of the distributions. We have shown the exploratory data analysis (EDA); including the descriptive statistics in table 1, the line, histogram, density, Normal Q-Q, Box-plot and ecdf plots in figure 4, empirical density and cumulative distribution plots in figure 5, we used maximum likelihood method to obtain model parameters, standard errors and p-value all appeared in table 2 and model selection criteria were used for comparison between the distributions (see table 3).

Min	1st Qut.	Median	Mean	3rd Qut.	Max	Skewness	Kurtosis
0.1000	0.6000	0.9000	0.8526	1.1000	2.0000	0.2722	3.5156

Table 1: Summary: Descriptive statistics of nicotine measurements data.

Parameter	LMWE Distribution			LMWER Distribution		
	Estimate	S. E	P-Value	Estimate	S. E	P-Value
	1.3354	0.0211	< 2e-16 ***	1.3268	0.0207	< 2e-16 ***
	0.0627	0.0002	< 2e-16 ***	0.1071	0.0005	< 2e-16 ***
	1.0594	0.0013	< 2e-16 ***	1.0611	0.0016	< 2e-16 ***
	-0.9285	0.0011	< 2e-16 ***	-0.9178	0.0013	< 2e-16 ***

Table 2: MLEs of the parameters for the LMWE and LMWR fitted to Nicotine data.

Model	AIC	AICC	BIC
LMWE	6676.86	6696.24	6697.24
LMWR	7958.67	7978.05	7979.05

Table 3: Model selection criteria for comparing the LMWE and LMWR Distribution.

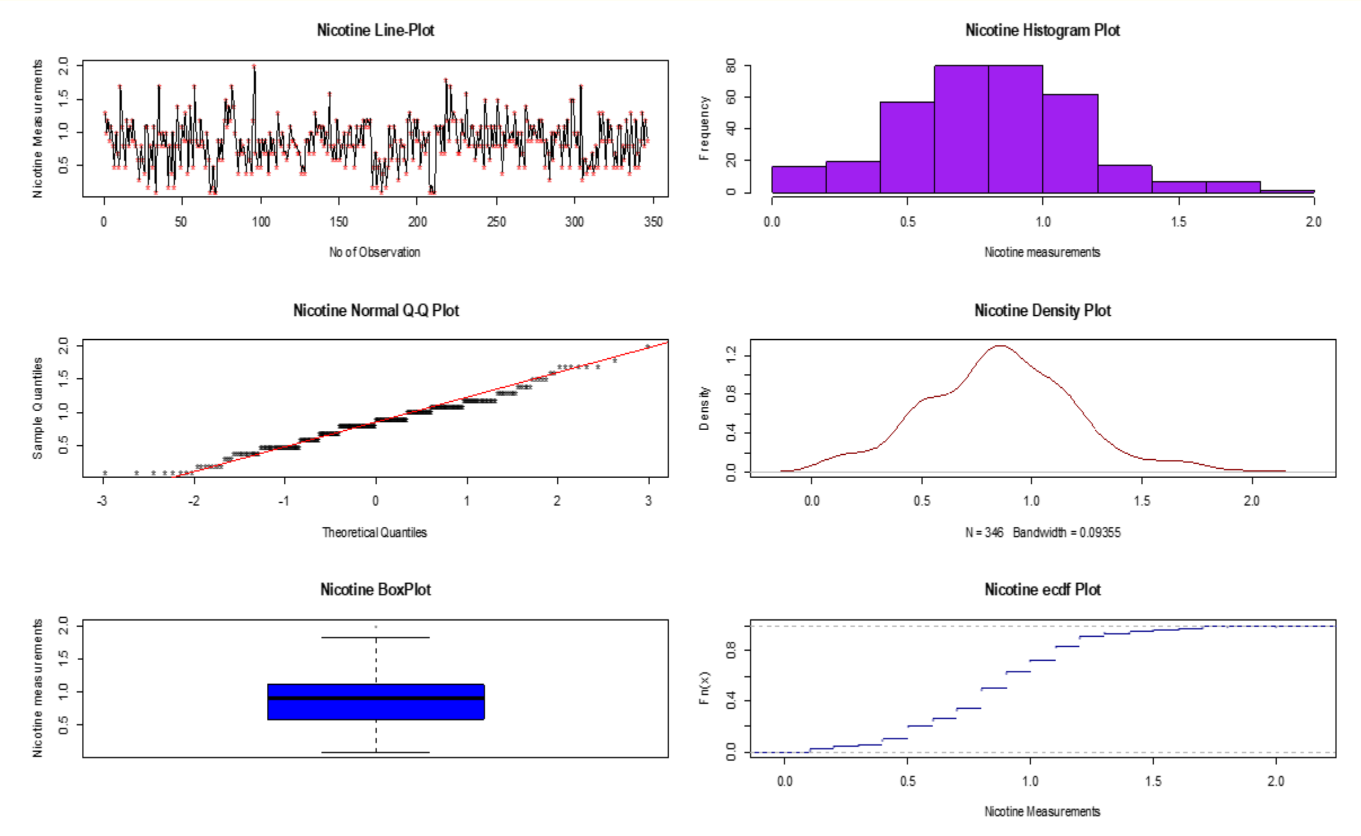

Figure 4: The line, histogram, density, Normal Q-Q, Box-plot and ecdf plots of Nicotine Measurement.

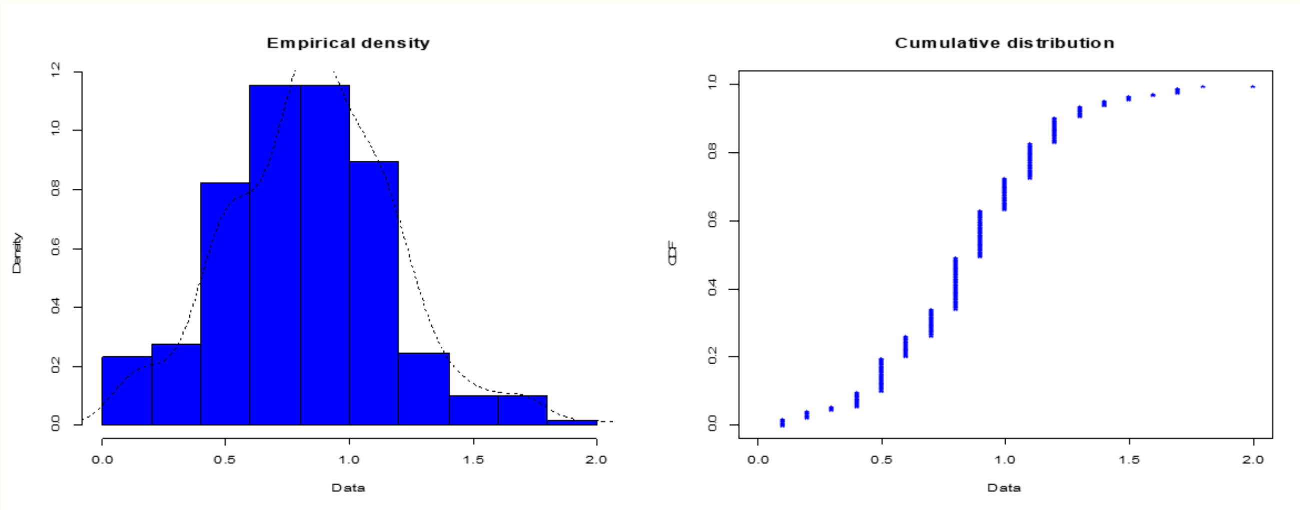

Figure 5: The empirical density and cumulative distribution plots of Nicotine Measurement.

Result and Discussion

Table 1, as we have mentioned earlier shows the skewness and the peak (kurtosis) of the data, (i.e the skewness and kurtosis of normal distribution is zero (0) and three (3)). But both skewness and kurtosis of the nicotine data used are more than 0 and 3, this fact made it non normal that led to more robust and flexible distribution. Figure 4 and 5 reflect the picture of the data. Values in table 3 are model selection criteria which enable us to compare the distributions; such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Consistent Akaike Information Criterion (CAIC), $Loglik_{LMWE}$=(-3336.428) and $Loglik_{LMWR}$=(-3977.333); and the values of LMWE are all less than values of LMWR as shown in table 3 above. Meanwhile, it implies that the output in both table 2 and 3 clearly shown that the LMWE distribution performed and more flexible (see figure 1 and 3) than LMWR distribution. Therefore, LMWE distribution provides better fit to the nicotine data than LMWR distribution.

Conclusion

In this study, we are able to establish some of the properties of the proposed distributions, for instance, the reliability function, hazard function, skewness, kurtosis, moments and generating function. The maximum likelihood method for estimating the parameters are also studied. Comparative data analysis and application of the proposed distributions is investigated considering nicotine data to reveals their performance and superiority, in which the LMWE outperform the LMWR distribution.

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