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Research Article

Two-dimensional Lattice Auxetic Structures Based on Square Unit Cells

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Abstract

The article explores the possibility of producing auxetic lattice structures using rigid square unit cells. An effective method of connecting unit cells to each other is proposed, the resulting structures are constructed, and the relationships - including the magnitude of linear expansion and porosity - associated with the transition of the structure from the closed to the open position are determined. This is related to the stretching procedure of the structure, during which the square unit cells undergo both rotation and displacement. By folding the planar structures one on top of the other, a 2D type volumetric structure has been produced exhibiting auxetic behaviour. Such layered structures exhibit synchronised movement when opening or closing, resulting in uniform and easily controlled deformations. On this basis, physical models have been built to experimentally confirm the relationships determined with geometrical models. A set of auxetic networks with enhanced stiffness constructed from rigid square unit cells in the form of solid squares and square frames has been proposed.

Keywords: Lattice Structures; Rotating Squares; Negative Poisson's Ratio

Introduction

Auxetic materials, structures, and fabrics (also referred to as 'auxetics' – usually an umbrella term for all of them) are materials that exhibit unexpected behaviour when subjected to mechanical stresses and deformations. When they are stretched in the longitudinal direction, they become thicker in one or more perpendicular directions. Also, in reverse deformation: when subjected to uniaxial compression, they shrink in one or more transverse directions.

Mechanical metamaterials in the form of cellular structures formed by the ordered arrangement of unit cells constitute a broad area of engineering interest. Above all, it is about producing a mobile structure with reduced weight while maintaining its high strength and stiffness. Such metamaterials can, at the same time, exhibit energy-absorbing capacity and also be suitable for thermal and acoustic insulation. In the case of auxetics, this goes hand in hand with a particular change in the linear dimensions of a metamaterial: expansion or compression in multiple directions simultaneously. By stretching the auxetic in the lateral direction, it becomes elongated in both that direction as well as in the vertical direction. Conversely, this phenomenon also occurs in compression– the auxetic material shrinks in both dimensions.

For the above reasons, both the engineering interest in the phenomenon of auxeticity and its usefulness in many areas are quite considerable. Auxeticity is continuously studied, and many efforts are being made for the development, design, and construction of auxetic structures. Most studies are done in the area of computational simulation, using physical models and computer graphics, e.g.: [1- 6].

The construction of physical models is also practised quite widely, yet even though the physical objects produced are indeed metamaterials, they often lack auxeticity [7, 8].

The most desirable studies, however, are those in which real and not just hypothetical physical models are built alongside computer modelling, which not only confirms the theoretical considerations but also encourages further application [9,10]. Feasibly designed macroscopic metamaterials are usually realised as demonstrators [9,10].

Many of these auxetic objects are produced by 3D printing techniques, referred to as additivemanufacturing [11,12]. It is expected that, produced with these techniques, auxetic metamaterials, especially lattice structures, will exhibit high elasticity and strength. To date, structures produced by 3D printing techniques are of insufficient quality, and their mechanical properties are relatively weak, e.g. in comparison with extruded products [12,13].

The existent applications of auxetics are quite limited as well. However, lightweight honeycomb or foam cell structures are already used in automotive, marine and aerospace structural solutions [14,15].

Lattice structures formed from unit cells can be used to manufacture sandwich panels, which are constructed from a metamaterial core in the form of a truss sandwiched between two rigid facings [23]. Typically, such a core takes the form of a structure made up of honeycomb cells or chiral cells, which are usually easily damaged – under certain stresses. Consequently, this utilitarian aspect is largely unexplored, although it is the subject of intense research [15-20]. As a result of mechanical action, even with a small change in their size, once the plasticity limit of the cell material is exceeded,the unit cells of the auxetic structure are crushed, and their ribs are broken. Numerous studies have examined this effect. What is particularly interesting is the search for a damage initiation criterion forelastoplastic materials [17-20].

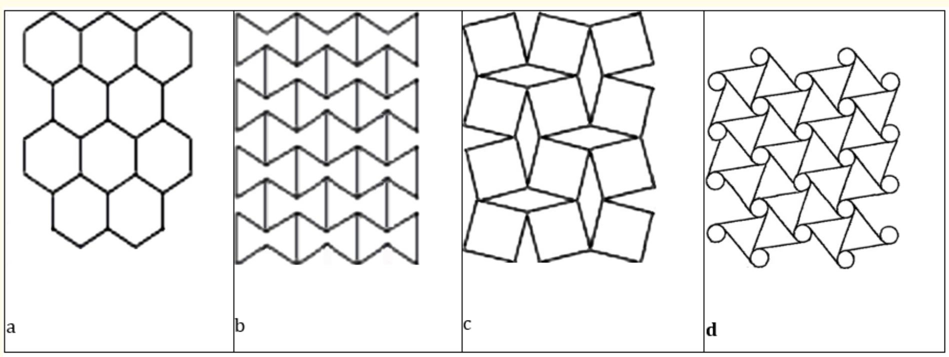


Figure 1: Basic unit cells assembled into auxetic structures: honeycomb cells, re-entrant' bow tie' cells, rigid square cells, chiral cells.

Identifying points of damage can lead to beneficial structural modifications [19], as by selecting materials with different stiffnesses [20].

Given the presented state of the research, we proceeded to build and analyse auxetic lattice structures based on square unit cells. The structures employ novel combinations of square unit cells, using axes of rotation near the corners of the squares [21,22].

The paper presents an efficient approach to the production of auxetic lattice structures, considering 2D planar structures as well as stackable and 3D spatial structures. The auxeticity of these structures, however, remained in two dimensions.

Planar auxetic structures

Mechanical metamaterials in the form of ordered planar structures are formed from planar unit cells such as honeycomb cells, re-entrant (‘bow-ties’), regular geometric figures, and also chiral units. The aforementioned types of unit cells are assumed to be the basic designs, although the possibilities for designing other, mostly similar unit cells seem to be endless. Nevertheless, unit cells in the form of geometrical figures of the re-entrant cell type (‘bow-ties’, rotational symmetry, triangular), chiral cells (chiral D, chiral twisting), and others such as Double-V, S-hinged, 4-star, missing-rib type, Y- shape etc. [3, 12].

The first two of these unit cells are strongly related, as they differ only in the angle sizes contained between the sections (ribs). Unit cells of the honeycomb type have all angles obtuse, while the 'bow tie' unit re-entrant cell has four acute angles and two obtuse ones. The honeycomb cell is of a convexshape, and the re-entrant cell (also called a 'butterfly') is of a concave shape.

It should be noted that among the unit cells shown in Figure 1, three of them: the honeycomb, the re- entrant cell, and the chiral cell, are made of straight bars or wires, and the squares originally proposed as surfaces can also be in the form of frames.

Unit cells linked together form structures that expand when subjected to tensile force. Such adeformation occurs at the hinges connecting the unit cells, e.g. the hinges connecting the squares - which in the existing solutions is dependent on the elastic properties of the unit cell material.

As a result of their interaction with each other and translational motion or simultaneous translational motion and rotation (squares), such structures formed from unit cells acquire auxetic properties. The structures shown in Figure 1b and 1c undergo contraction in the transverse direction in compression, while another structure – shown in Figure 1d – undergoes contraction due to the twisting of the chiral cells. Although the structure shown in Figure 1a is included in this group of unit cells, it actually extends in the longitudinal direction during transverse compression or collapses. In general, the structures change their shape, i.e. they are mechanically reconfigurable and may additionally exhibit energy absorption or surface masking [23].

The auxetic structures shown in Figure 1 are usually enlarged by further unit cells, forming large surface planar structures, which can have a number of applications, e.g. in engineering structures, sports equipment or medical products [24-25]. The manufactured structures of periodic artificial elementary cells at the mesoscale yield the desired macroscopic properties - in this case, auxetic properties, i.e. a Negative Poisson's Ratio (NPR) of -1.

The auxetic behaviour of this type of structure, especially under tensile force, is determined by the elastic properties of the connections between the unit cells. These very connections are the weakest spots of such structures, as they are subject to critical stresses that usually lead to their failure. This genuine problem is usually overlooked in elegant computer simulations [26-29]. In addition, many theoretical analyses of auxetic structures do not deal with entire structures but with individual unit cells, which makes these analyses of little practical use.

Structures made of rotating squares

The auxetic structure shown in Figure 1c was popularized by Grima [27,28] and is regarded today as a standardsolution. This is an idealised model in which the non-deformable squares are free to rotate within a certain small angular range.

In the closed position, the sides of the squares are adjacent to each other so that they form an enclosure. As a result of the tensile force, an unusual effect of simultaneous expansion in the vertical and lateral directions is observed for them - resulting in a negative Poisson's ratio NPR of -1. In practice, the functioning of such a structure is very limited, as the connections of the squares are easily destroyed.

An arranged collection of squares of identical size, specifically joined at the corners, already constitutes a canon of auxetic materials. When the tensile force is applied, the rigid squares interacting with one another not only rotate but also move, resulting in a change in the size of the structure. Because this change in size occurs simultaneously in both the lateral direction and the vertical direction (when force is applied only in the vertical direction, for example), an NPR effect (and a spectacular visual effect) is obtained – as shown in Figure 2.

By modifying these structures by placing the axis of rotation on the surface of the squares (near their corners), a failure-free auxetic structure is obtained. In this simple way, the squares achieve excellent mobility, although in this case, the elastic properties of the square unit cell material and their connections (i.e. the hinges) cannot be utilised. The novel solution used, Figure 2b, requires the application of force (stimulus) in both tension and compression.

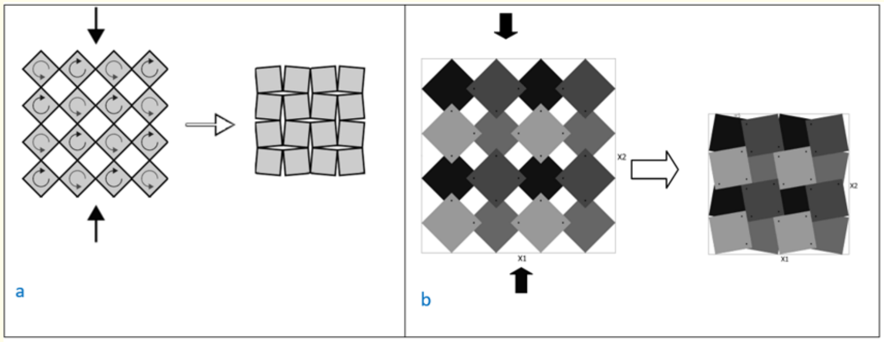


Figure 2: 4 x 4 structures of rotating squares similar to those proposed by Grima and Evans [27] -a and in a modified form [21] -b.

While in the solution shown in Figure 2a, the structure of the rotating squares in compression acquires a certain amount of potential energy, allowing it to return to its original form once the force ceases. In contrast, when it is stretched, the structure, due to the elasticity of the material, undergoes tension and also returns to its initial state when the force ceases. It can be seen from the above that, in this case, the elastic properties of the square cell material play just as important a role as the structure itself.

In the applied modification, however, the use of the elastic properties of the material of the squares and hinges is not possible, and the structure is subject to fixed dimensional changes when external forces are applied (actuator mechanism). In this case, two stable end positions of the structure can be distinguished, i.e. the closed position and the open position (Figure 2b). Taking into account the relative size differences of the constructions corresponding to these positions, one obtains their ratio, which corresponds to NPR, also with a value of -1.

Analysis of geometric relationships in a full-square structure

Squares as unit cells connected by axes of rotation on their surface form 2D auxetic structures. By analysing the geometry of the structure in the closed and open positions, a number of functional relationships have been established. The decisive parameter, in this case, is the distance of the axis of rotation from the edge of the square, as shown in Figure 3.

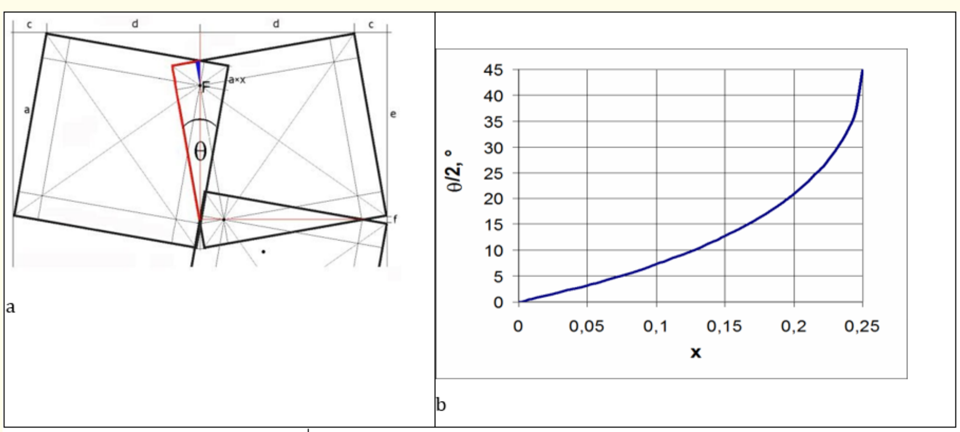


Figure 3: Two squares connected by an axis at point F (a) and the functional relationship between the theta angle and the parameter x (b) [21].

We define the parameter x as a dimensionless quantity that, when multiplied by the side length of the square, gives the distance of the axis of rotation from the edge. The value of the parameter x varies between 0<x<0.25.

In study no. [21], it was shown that between the theta angle (the inner angle between the edges of the squares in the closed position) and the parameter x, there occurs the following relationship:

$$\tan\frac{\theta}{2} = \frac{1+\sqrt{(1-4x)}}{1-\sqrt{1-4x}}$$

As can be seen from Figure 3b, the θ/2 angle reaches a value of 45° for x = 0.25. Then the squares block each other, and the structure remains still because the squares can no longer rotate.

For the parameter x = 0, the axis of rotation moves to the end of the corner of the square, and in the structure's closed position, the edges of the squares are in exact contact with each other (Figure 2b).

The x parameter is therefore crucial for the formation of the structure, and the position of the axis of rotation in the rigid square cells leads to the resulting shape and the maximum size of the structure in the open position, with the collective behaviour of the structure always yielding a Poisson's Ratio of -1. The transition from the closed to the open position results in an expansion of the structure, with an increase in its size both horizontally and vertically as a result of the applied force.

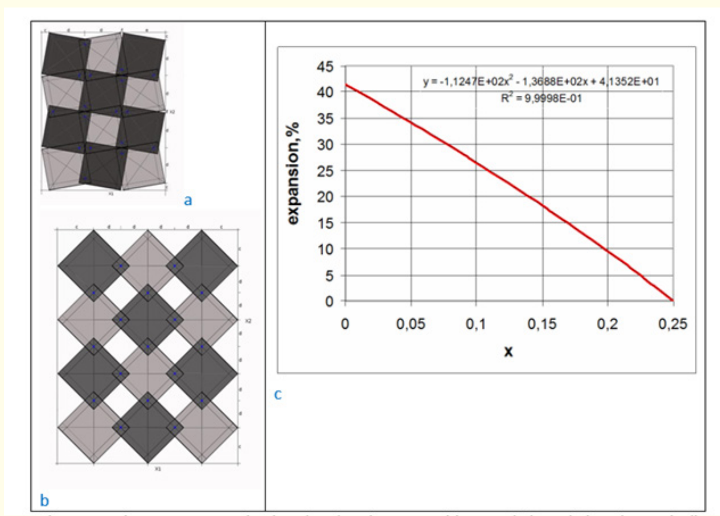


Figure 4: Auxetic structure 3x4 in the closed and open positions and the relative change in linear dimensions (c).

As for the uniaxial tensile load in the plane, the impact is transferred from the cell through the axesof rotation located at the corner to the adjacent cell.

The connection of the squares can be described by parameters such as the length of the side of the square a, the internal angle theta in the closed position (Figure 3a) and the geometrical parameter x. The a parameter represents the total length of the square, theta is the angle between the edges of the squares in the closed position, and x is the geometrical parameter where a´x is the distance ofthe axis of rotation from the edge of the square. The total length of the structure is X1. It can be calculated by establishing the analytical relationships for the individual sections c, d, e, and f. (see Figure 4a).

Figure 4a shows the values of the individual sections of the structure that make up its X1 and X2dimensions. From the geometric analysis, these values can be calculated exactly.

closed	$a * \sin\frac{\theta}{2}$	$\frac{a * \cos\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$	$a * \cos\frac{\theta}{2}$	$2a\frac{\tan\frac{\theta}{2}\sin\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$
open	$\frac{a}{\sqrt{2}}$	$\frac{a*(1-2x)}{\sqrt{2}}$		

Figure a: Lengths of sections making up the dimensions of the structure [21].

For the closed position, there is a strong relationship between the lengths of the sections making up the overall dimensions of the structure and the size of the theta angle. In the open position, on the other hand, the dimensions of the structure are determined by the length of square side a and the x parameter.

Given the above relationships, it is possible to determine the sizes X1 and X2 and their changes associated with the transition from closed to open positions for each structure of this type.

The analysis of the change in the dimensions of the structure between closed and open positions shows that, for a given value of the parameter x, the relative change in linear dimensions (expansion in tension) varies from a value of 41.42 % for x = 0 to zero for x = 0.25. The value of the expansion of the structure decreases along with the increase of the parameter x (Figure 3b). Analysing this structure, we found that the relative value of the expansion does not depend either on the size of the square side or the number of the elements and is only a function of the parameter x.

One of the most important parameters for lattice structures is the spacing between the nodes, in thiscase, between the axes of rotation. These gaps change when the structure is stretched from the closed position to the open position.

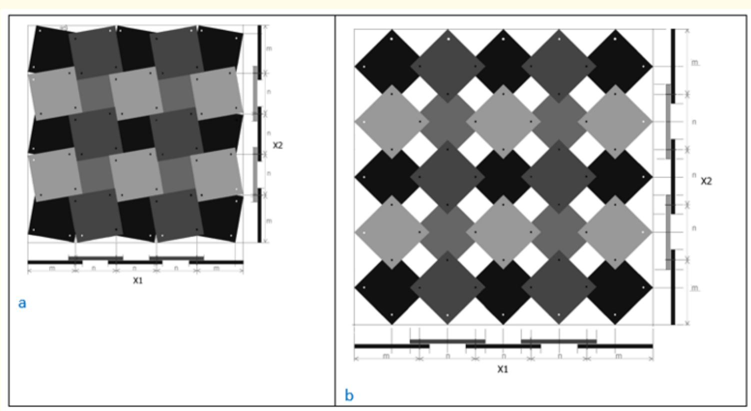
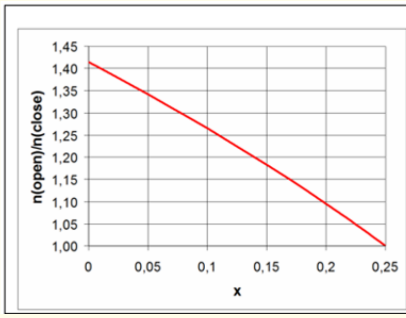


Figure 5: 5x5 auxetic structure in the closed and open position with marked m and n distances between the axes of rotation.

For the presented 5×5 structure, with the parameters x = 0.125, θ/2 = 9.7°, the n(open)/n(close) ratio is1.22, i.e. in tension the axes move 22% horizontally and vertically.

This follows from the condition: n(open)/n(close) = m(open)/m(close). Figure 6 shows the graphical relationship between the n(open)/n(close) ratio and the parameter x. The significance of this relationship is that it highlights the fact that the mechanism of modified auxetic structures based on rotating square unit cells involves both rotational and translational motion.

The relationship shown indicates that the axes of rotation in tension get increasingly displaced as the parameter x gets smaller, i.e. the closer the axis of rotation is to the square's edge.



see fig.5

m(close)	$a(1-x)(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})$
m(open)	$a\sqrt{2}(1-x)$
n(close)	$a\sqrt{2}(1-2x)\cos(\frac{\pi}{4}-\frac{\theta}{2})$
n(open)	$a\sqrt{2}(1-2x)$

where a is the side of the square
x geometric parameter

Figure 6: The ratio of the distances between the axes of rotation in the auxetic structure as a function of the x parameter and the relationship of the lengths of the distances between the axes of rotation n and m.

Another value characteristic of the auxetic structure under study is the total area covered by its outline, i.e. the sizes X1 and X2.

The analysed structure covers an area that can be calculated as the product of the lengths X1´X2. This value changes with the transition from the closed to the open position. Regarding the ratio of the outline area of the structure in the open and closed position, it can be shown that its value decreases along with the increase of the x parameter, in accordance with the relationship below:

$$\frac{(X1*X2)_{open}}{(X1*X2)_{close}} = 2(1-2x)$$

By moving the axis of rotation of the squares further away from their edges (more overlapping areaof the squares), the outline area of the structure decreases.

This is a direct relationship between the outline area of the structure and the geometrical parameter x, which is obviously related to the change in the linear dimensions.

The surface area of the resulting pores when the structure is opened is also a function of the parameter x. In the open position, each pore covers an area equal to $(a-4ax)^2$. For a parameter value of x = 0.25, when the structure becomes still, the pore area is equal to zero.

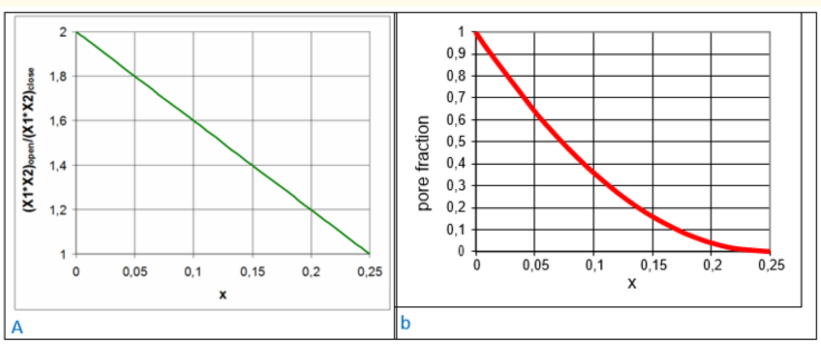


Figure 7: Relative change in the area of the structure outline between the closed and open positions as a function of the x parameter (a) and relative change in the area of the pores in the open position as a function of the x parameter – for full-square structures (b).

It can be seen from the relationship in Figure 7 that there is a large change in the pore area in the open position if the axis of rotation shifts.

This area may be subject to less change if frames (squares with their interior cut out) are used instead of full squares. This option is discussed below.

Another special property can be identified for the modified design of rotating squares. In compression of the structure, the transmission of stress occurs first at the axes of rotation and then, when it reaches the closed state, at the contacting edges.

In the closed position, parts of the squares' edges meet (Figure 5a). This transfers the force acting during the compression of the structure. With an increase in the applied force, the structure remains stable as long as the contact stiffness of the square unit cells is not exceeded. As can be seen in Figure 5a. the length of the contact areas between the square units in the closed position is less than twice the distance of the axis of rotation from the edge, k>2a´x. Analysing the geometrical relationships in the structure, one can establish that as the parameter x increases, the contact length k increases.

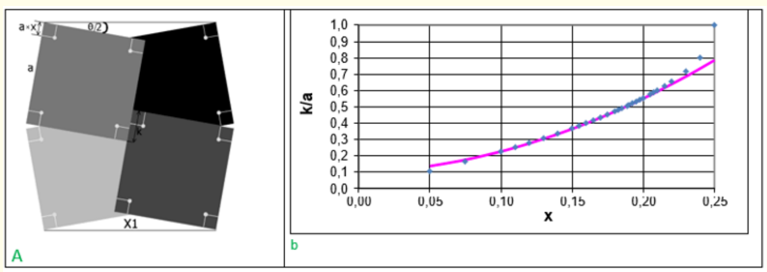


Figure 8: 2x2 auxetic structure in the closed position with x and k marked (a) and the functional relationship between the k/a value and the parameter x (where a is the length of the square side).

The approximate function curve is of the form:

$$\frac{k}{a} = 9{,}592x^2 + 0{,}366x + 0{,}095$$

It should be noted that the formula (2) does not satisfy the boundary conditions, although it can be very useful for physical structures.

Geometrical analysis of lattice structures

Square-frame planar structure

By replacing the flat solid squares with square frames, a new lattice structure is obtained, with properties similar to solid square structures.

Figure 9 shows a 3´3 structure constructed of square frames in two extreme positions: closed and open. It should be noted that in the closed position, the edges of the squares come into contact, and the minimum porosity of the structure is reached.

The opening of the structure when it is stretched does not lead to a deformation of the square frames but to an increase in the total pore area. The pore-opening properties are characteristic of this particular auxetic structure.

In the closed position (for low values of the x parameter), one type of pore is present – in the form of squares of side b. In the open position, on the other hand, the presented auxetic structure exhibits two types of pores (excluding the extreme squares) – marked in Figure 9 as A and B. The first type (A) is of area equal to $(a-4ax)^2$ (the same as for the solid squares), while the area of the second one (B) depends (in this case) on the size of the cut-out b.

For higher values of the parameter x, the B pores become octagons formed from squares with side b. The area of such an octagon depends on both the b dimension and the x parameter.

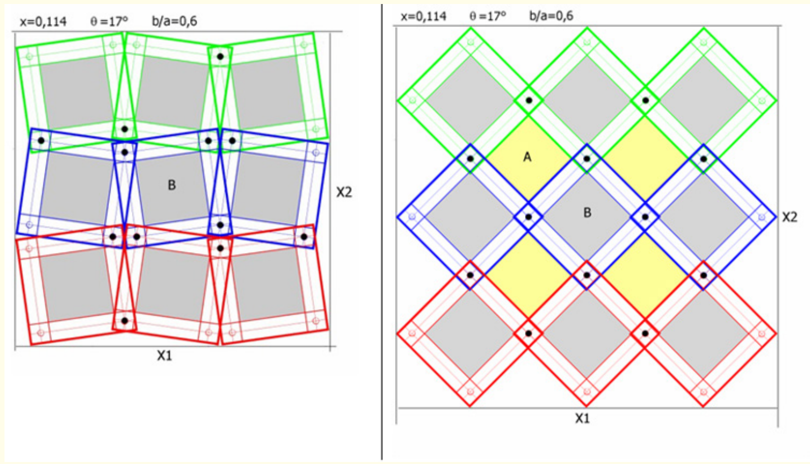


Figure 9: 3x3 auxetic structures built from frames with a given cut-out in the closed position and in the open position.

In the closed position, the structure made up of square frames has one type of pores – the B type, i.e.those coming from the square cut-out. The second type of pores – A – appears in the open position.

The problem of the shifting axis of rotation when the structure is stretched can also be traced back tothe changing number of pores.

Figure 10 shows the arrangement of pores in a 6´2 structure formed from frames with parameters x = 0.125, b/a = 0.6. The B pores have an area approximately equal to b^2. The area of the A pores, on the other hand, is a function of the parameter x and is $(a-4ax)^2$. The A pores are absent from the closed position containing only the B pores.

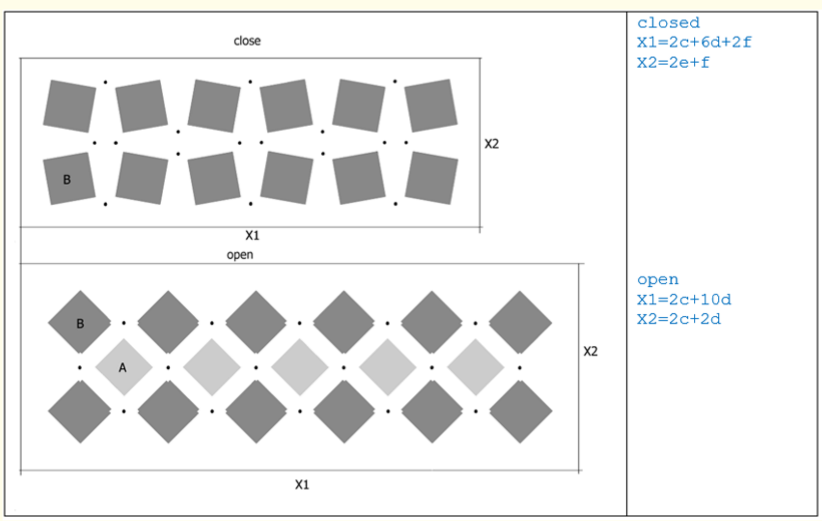


Figure 10: Pore layout for a 6x2 structure formed from square frames in the closed and open positions.

Figure 9 also shows the position of the axis of rotation and the overall dimensions of the X1 and X2 structures. Based on this, one can comprehensively analyse the following: the change in the linear dimensions of the structure, the displacement of the axes of rotation, the change in the size of the pore area, and the total area of the pores, as well as their number. The transition from the closed to the open position is related to the expansion of the structure and the movement of the rotation axes.For a parameter value of x = 0.125 and θ = 19.5°, the relative change in dimensions in the X1 direction and in the X2 direction (expansion) is equal to 22.47%, the displacement of the axis of rotation is 22%, and the ratio associated with the change in the surface area is equal to (X1*X2)open/(X1*X2)closed = 1.5.

The diagram demonstrates that, for a structure made up of square frames, a change in the size of the'empty area' is associated with a transition from the closed to the open position. In the open position,in addition to the B pores on the inside – formed from the cut-out of the frames, there are A pores between the frames.

In the example analysed, the increase in the total pore area when moving from closed to open position is 28.94%.

This indicates that by using square frames instead of full squares, the size of the 'empty area' for the open position was significantly increased. It also follows from the above that by introducing a cut-out inside the squares, the size of the empty area can become larger so that the porosity of the structure can be significantly increased, with the Poisson's ratio remaining constant at -1.

It can be calculated that the empty area, i.e. the total area of the pores for the 6´2 structure presented in the closed position, is no more than 38.98 %, while in the open position, it constitutes 33.5 % of the total surface area, i.e. X1*X2.

This new design of rotating frames with the axes of rotation within their planes achieves an NPR effect, which for the difference in the size of the structure between the 'closed' and the 'open' position is equal to -1. This pertains to the auxetic effect under uniaxial load.

A planar structure from rigid bars

A new variant of frames can be an arrangement of bars (column structure) of the same dimensions, which can be connected by rotation axes. In this case, each of the frame elements can rotate freely so that a lattice structure resembling an auxetic structure can be produced.

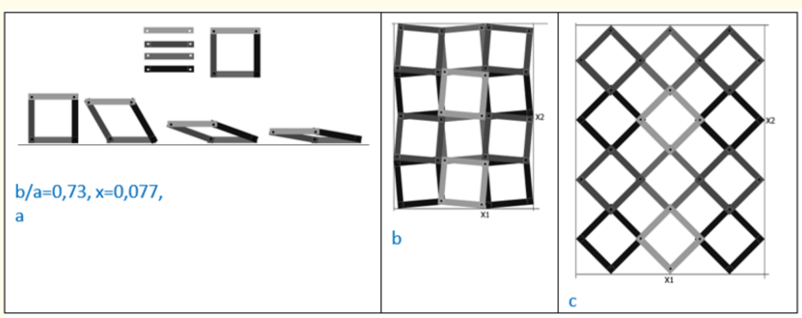


Figure 11: Elements forming a square (a) and possible column structures in the closed position (b) and in the open position (c).

It can be noticed that in the column structure produced in this way, there is only a local interaction ofthe components but not of the square unit cells. If the sides of the squares become mobile, then the structure loses its functionality because the displacement of the elements of the squares does not have the effect of moving the neighbouring elements.

This is a fundamental obstacle to the realisation of an auxetic structure, as the components of the unit cells have a certain stiffness, their combination allowing for a hinged movement, yet not for the predetermined transmission of stresses, but only randomly.

As can be seen from the above considerations, it is, therefore, necessary to use whole rigid unit cells rather than their moving components to obtain an auxetic effect.

A volumetric structure

Planar lattice structures made of rigid square frames can be transformed into a volumetric structure if they are stacked on one another. By extending the axes of the rotating squares, a new spatial structure is obtained. This requires planar structures to be connected to each other. In engineering terms used in this area, the individual bottom chords and top chords of the truss will be connected toeach other using bars.

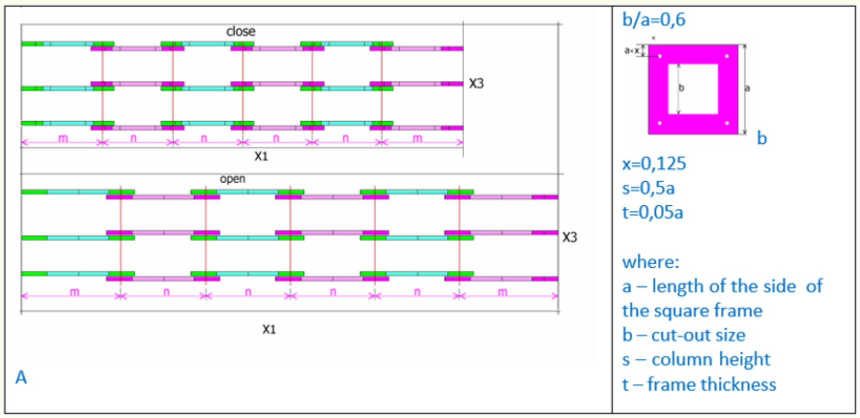


Figure 12: Three-level 6x2x3 lattice structure with square frames in the closed and open positions (side view) and a single square frame along with its parameters.

The diagram in Figure 12 shows the three levels of the square frame structure with the marked axes of rotation which also constitute the bars of the structure. As a result of stretching the structure, the distances m and n between the axes of rotation increase, in this case, by 22.47 %.

The X1 and X2 dimensions of the planar structure can be calculated from the formulae provided in Tab. 1. The X3 dimension, on the other hand, is the sum of the thickness of the frames and the lengthof the bars X3 = 6*t+2s, where t is the thickness of the frame, and s the dimension of the bar. As a result, the volume of the spatial lattice structure is X1*X2*X3. The expansion of the volumetric structure between the closed and the open position will correspond to the ratio of the planar surface in these positions and will be, in this case (6*2*3) matching the expansion of the surface area, i.e. 50%. This quantity is strongly dependent on the x parameter (with assumed values of b, s, and t).

For the volumetric design shown, the change in size is apparent only in the X1 and X2 directions, and therefore NPR = -1 applies only to the surface.

An important parameter of a volumetric lattice construction is the empty space. In this case, it is the difference between the volume of the outline X1*X2*X3 minus the volume of the frames $n*(a^2-b^2)*t$ and the volume of the bars $4n*s*\pi*2t^2$. For the example analysed (Figure 11), in the closed position, the empty space is more than 87%, while for the open position, more than 91% of the volume of the X1*X2*X3 outline.

A stack of planar auxetic structures may be proposed for the core of sandwich panels – consisting of two external surfaces (the panel's skin) and a low-density core. Numerous proposals for sandwich panels with different core geometries are known [29], but the predominant ones are those withcores based on structures composed of re-entrant "bow-tie" unit cells [30,31]. These solutions exhibit good mechanical properties but very little dimensional change – very little shrinkage in compression [15].

If a system of planar auxetic structures based on unit cells of modified rotating squares is used as the core, a much bigger size change can be obtained.

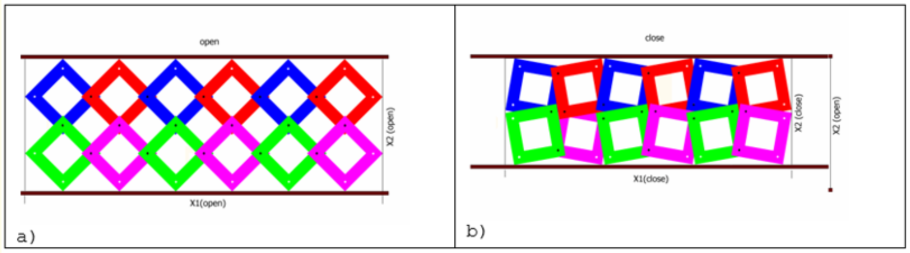


Figure 13: Proposed sandwich panels with a core of modified rotating squares in two auxetic structure positions.

From the diagram shown (Figure 13), it can be seen that a significant dimensional contraction occurs as a result of the compression of a panel with such a core, which in this case, for the parameter x = 0.125 is 20%.

It should be added that planar structures set vertically must be appropriately spaced so that they all are in a vertical position. The core will change size in compression in a direction perpendicular to the face of the panels. This idea requires, in practice, additions to the fixing of the core and the addition of a mechanism for its mobility – enabling the transition from the closed the open position (Figure 13).

The experiments

Flat physical lattice structures

By creating a negative from the solid rotating square structures, flat lattice structures built from frames have been obtained, achieving auxetic properties provided that the elements forming them had a certain stiffness. This property allows unit cells to effectively interact with each other and enables auxetic behaviour.

The lattice structures from the negative of the full rotating squares, i.e. square frames, were made using a 3D printing technique.

This experiment involved rigid square frames connected by axes of rotation. The model provedeffective in obtaining auxetic properties.

The introduced axes of rotation constitute a kind of hinge – as previously shown for full squares [**21**]. This way, the introduced modification in the basic elementary cell of the auxetic structure wasintended to eliminate local stress concentrations during deformation. In an auxetic structure, the position of the axis of rotation on the surface of the elements (squares or square frames) ensures its proper functioning and transmission of loads and deformations.

By supplementing the frames with rotating axes and combining them into a planar structure, a new type of auxetic structure was obtained. Due to the presence of symmetry axes in the structure, an in- plane expansion was obtained – with similar values in the x-axis direction and in the y-axis direction, indicating the isotropy of the continuum. These experimental considerations concern this feasible structure.

In a version of the structure modified in this way and using the connection of rigid frames with axes on their surface, a lattice structure was created. This yielded a new type of 2D porous auxetic material.

The resulting lattice structure consisted of rigid frames forming unit cells connected with smooth hinges in the form of pins.

For this structure with parameter x = 0.0375, an expansion in the horizontal direction of 34.88% and inthe vertical direction of

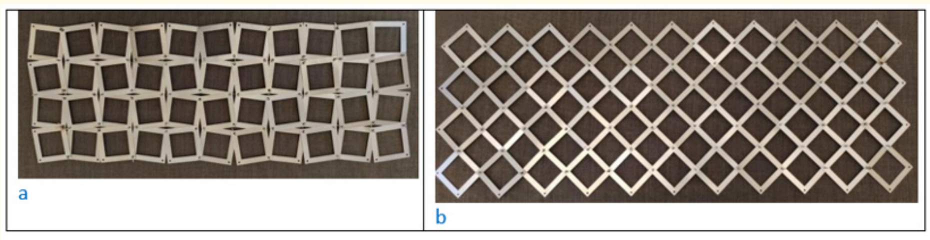


Figure 14: 11x4 auxetic planar lattice structure of aluminium frames, in the closed position (a) and in the open position (b).

34.34% was obtained in tension, while the theoretical value was 36.05%. These differences are due to the imperfect closure of the structure.

As a result of the tensile forces, rotation and displacement of the unit cells occur – leading directly to simultaneous expansion in a direction perpendicular to the tensile forces. It should be noted that the rotation of the frames is also accompanied by their displacement, resulting in an expansion of the structure that leads to a theoretical NPR value of -1, and in practice, there is little deviation from it.

Expansion cannot be initiated internally but requires the application of an external force. In order to achieve an open position and increase the size of the structure, it is necessary to pull its edges. Also, to fold the structure into a closed position, a similar force needs to be applied. Such a change in the dimensions of the structure can, in this case, occur slowly in a uniform manner or rapidly in a shock mode. For such structures, it has been observed that as the number of unit cells increases, partial inertia effects in the deformation dynamics appear. Nevertheless, the functioning of these modified auxetic structures is not compromised. The presented modified lattice structures with auxetic properties with units in the form of square frames, connected to each other with rotation axes on their surface, allow the square unit cells to rotate without damaging the structure, which can be repeatedly stretched and compressed. As far as the possibility of failure of such a lattice structure is concerned, in the closed position it is secured by the structure material's compression resistance, while in the open position – by the properties of the rotation axes elements, and to a lesser extent,by the tensile resistance of the material.

An indispensable mechanical property of the square frame material is its rigidity. The material of the rotation axes, however, apart from being rigid, must also be tear-resistant.

This means that two factors are responsible for the auxetic functionality of the lattice structures under consideration, namely the rigid square frames as unit cells and the introduced axes of rotation.There is, therefore, no need to consider the elastic properties of the frame base material in this solution, only its resistance to deformation. The tested lattice structures formed from frames in the form of cut-out squares featured NPR values of -1, regardless of the size of the pores, i.e. the proportion of empty spaces in the structure.

Multi-layer physical lattice structures

This part of the research results presents a new design of rotating squares and the possibility ofproducing multilevel 2D auxetic structures from them in the form of stacked planar structures.

By stacking planar structures made of rigid frames on one another, it is possible to obtain avolumetric lattice structure able to expand in the planar range.

Structures assembled from square metal frames connected with screws are shown below.

Figure 15 shows a volumetric lattice structure formed from planar structures consisting of square- shaped steel frames (a = 40 mm, a´x = 4 mm, b = 22 mm, t = 0.5 mm). On each of the three levels, the volumetric design contains nine gaps (cut-out squares).

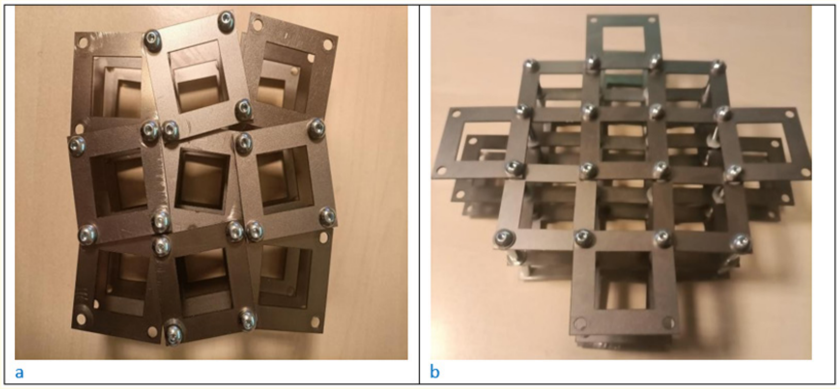


Figure 15: Three-level 3x3x3 flat structure in the closed position (a) and in the open position (b).

The measured values for the change in linear dimensions between the closed position and the open position are: X1(close) = 116mm, X2(close) = 115 mm, and X1(open) = 144 mm, X2(open) = 143 mm. This leads to an NPR (Negative Poisson's Ratio) for the whole structure, albeit only in the X1-X2 plane. The structure's height of 45mm remains unchanged.

It is understood that lattice structures in the form of a stack of planar structures can be extended both horizontally and vertically – Figure 16.

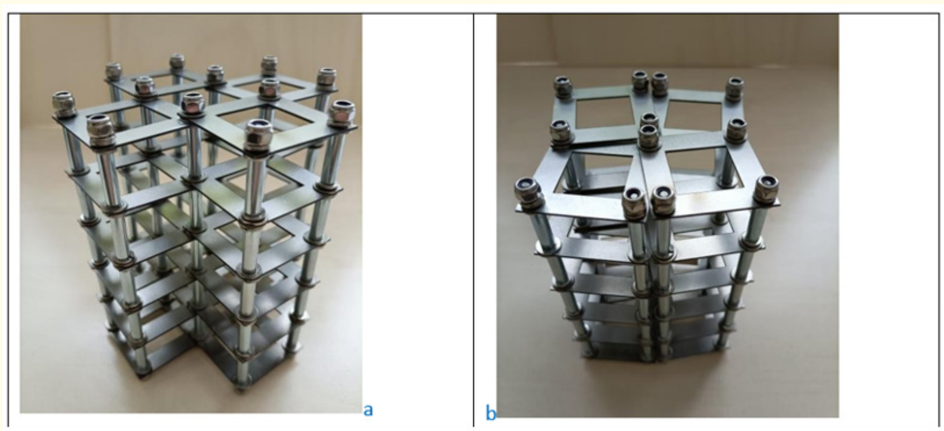


Figure 16: 2x2x6 planar structure in the open position (a) and in the closed position (b).

The extension of the structure in the vertical direction involves extending the rotational axes of the individual squares. Using square frames with a side length of a = 40 mm, a cut-out of side b = 22.5 mm, and a parameter x = 0.1, a theoretical expansion of 26.53% is expected. In this case, the linear dimensional changes measured between the closed position and the open position are: DX1 = 21 mm, DX2(close) = 20 mm, (with a height of 111 mm), yielding a linear expansion of 25.91% in the X1 direction and 24.69% in the X2 direction. This leads to an apparent NPR of -1.05.

The trusses produced, due to a large amount of friction contact, already showed a noticeable frictional effect, even though additional shims were used to reduce it. It can be assumed that an ideal truss will have no appreciable frictional effect, with zero in-plane bulk modulus and unlimited shear moduli.

The dominant deformation mechanism of the truss in the transition from the closed to the open position was axial deformation: the rotation and translation of its components - the square frames. By adjusting the construction parameter values, such as the b/a ratio and the x parameter, it is possible to determine the amount of empty space in the feasible construction.

One can expect that through multilevel auxetic structures, it will be possible to build adjustable and durable mechanical structures with a reduced mass.

Pictured below (Figure 17) is a prototype of a sandwich panel showing a large change in dimensions in the vertical plane between open and closed positions.

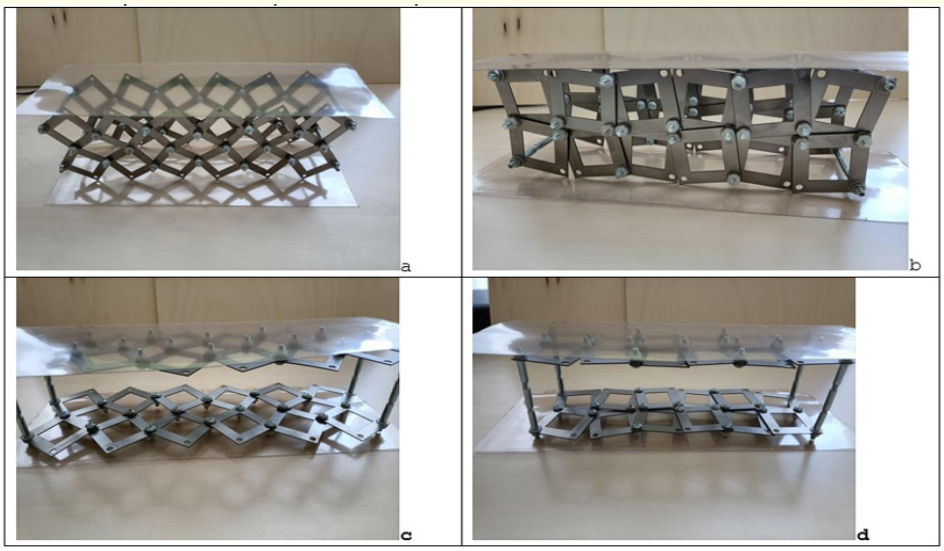


Figure 17: Photographs of auxetic structures making up the core of sandwich panels in different arrangements.

The auxetic structures proposed above have important and useful features, namely a large empty space volume and considerable mechanical strength (metal structure). They thus retain the required mechanical properties, which play a key role in solid cell systems.

Discussion

Multilevel auxetic structures are structures obtained by stacking successive layers of flat structureson one another and joining them with bars.

Many researchers have already studied and developed the basic structural innovations in this area. Examples of constructing three-dimensional lattice metamaterials formed by combining unit cells are present in numerous papers [23,32-39]. A number of 3D architecture structures have already been created from polymers, metals and composites, especially using incremental manufacturing technology. Their 3D architecture was obtained by inserting load-bearing ligaments between the unitcells. These, however, are still a weak spot of all these solutions, as they tear when critical stresses are reached.

Such structures can also achieve negative Poisson's ratio values, either along the plane of the component planar structures made of flat unit cells or in all directions in the case of volumetric unit cells.

Birman and Kardomateas [35] and also Feng [23] have reviewed the modern trends in theoretical solutions, novel designs and modern applications of sandwich structures, covering hundreds of articles on the subject. Driven by the application aspect, the authors of these numerous works investigated auxetic lattice structures as cores for sandwich panels [36].

The research in these works mainly focused on demonstrating the favourable mechanical properties of sandwich planes containing auxetic structures at a reduced mass. These include the increased strength [40] demonstrated in compression tests [41,42], impact tests [42,43], bending tests [44], aswell as energy absorption capacity [17,45-50] and thermal insulation [50].

The work presented herein focuses on demonstrating the effect of linear expansion in multilevel lattice auxetic structures. According to the theoretical analysis, a significant expansion of the structure was obtained in tension, which, depending on the value of the geometrical parameter x, is in the range of up to 41.42%. In the works cited above, this figure is much lower, amounting to only a few per cent [48,49].

The volumetric lattice structure presented here, made of metal frames, showed an expansion which, in tension, occurs only in the horizontal plane, reaching more than 20%.

The structures proposed here, exhibiting auxetic behaviour, do not have an energy absorption effect, as the movement of their elements is not related to utilising the elastic properties of their material. Moreover, due to the lack of stresses, the functioning of the structure is purely mechanical.

It appears, however, that such structures can also be used to absorb kinetic energy, as the energy of the irreversible deformation of the structure and the transition from the open to the closed position, e.g. with adjusted transition resistance. What is beneficial in this case is that large deformations are possible. The proposed lattice structures exhibit predictable responses to a given loading condition. The opening of the structure from the closed down to the open position comes with a relatively large linear expansion.

It is particularly noteworthy that the assembled auxetic structures (physical models), both planar andmultilevel ones, based on square frames exhibited reliable performance.

While the relationship between such structures and their mechanical properties is not known, it can be expected to be favourable, given the higher stiffness of the structure.

Here as well, forces that cause stretching in a given direction lead to changes in size in the direction perpendicular to the principal stress, which may, in practice, make such multilevel structures more resistant to indentation because the acting stimulus will be utilised to change their size.

From the continuum point of view, this metamaterial can be described as a material with cubic symmetry characterised by three geometrical parameters (length, width, and height), although their auxetic properties are determined by planar symmetry. In this kind of symmetry, both the square elementary cell and the cell connections play a major role. The size of the elementary cell is determined by the size of the square frame, i.e. by its length and the size of the cut-out.

A stack of 2D planar structures made of rotating squares is a new spatial structure that, in general, consists of unit cells joined by axes of rotation and supports as their extensions. Supports (bars) connect the planar levels. It is thus a combination of metal and empty space, just like in a typical 3D structure. The proportion of empty space in a 3D structure is determined by the empty space factor, and its variation is due to the elongation of the structure along its plane. This elongation can be traced by changing the position of the unit cell (square frames) rotation axis.

The lateral deformation of this structure will be positive under uniaxial tension and negative under uniaxial compression.

This effect seems particularly interesting for sandwich panels in which the cores can be made of an arrangement of planar auxetic structures composed of rotating square frames.

We can conclude that this work has presented the construction of large-scale metamaterials through the precise assembly of a large number of square frames. It is shown that a continuum can be achieved by combining rigid parts and their connections. The position of the connections in the form of rotation axes influences the size changes obtained, each time ensuring an NPR of -1, i.e. a negativePoisson's ratio effect without damage to the structure.

This way, new types of auxetic trusses have been demonstrated for which the geometrical relationships are established by positioning the axes of rotation on the surface of the frames.

The presented lattice structures were made in the same way, as they were composed of square frames with axes of rotation, allowing for a highly repeatable process of producing mechanical metamaterials on a macro and micro scale.

These advances also indicate the importance of the material aspect of auxetic structures. In the physical (real) world, as opposed to the digital (virtual) world, material constraints are intrinsic to their form and behaviour, and the defined geometry of the system must depend on them.

Conclusion

Obtaining a lattice volumetric auxetic structure with unit cells in the form of rigid square frames interconnected to form planar structures was made possible by connecting them bars. This system can be treated as gradient-free since its mechanical properties depend on the topology and, in extreme cases, on the stiffness and strength of the material, primarily the material of the rotation axes.

The two configurations, i.e. the closed position and the open position, show that it is possible to design the system (for a given value of the x parameter) in such a way as to achieve the required degree of longitudinal and transverse elongation and that these positions can be reached any number of times without damage.

The relative elongation of the structure is not sensitive to the size of the square cells or the numberof frames in the structure.

Modified designs based on square unit cells result in an NPR of -1, which is independent of the elasticproperties of the structure material.

Credit Authorship Contribution Statement

J.P.: Conceptualisation, Methodology, Design, Formal analysis, Investigation, Writing – original draft, Visualisation, M.P. Supervision, Funding acquisition

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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