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# Short Notes on the Rotatory Motion of a Rigid Body

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## Abstract

In this short study, light is shed on the rotatory motion of a rigid body about a fixed point under the action of different external fields and moments. These fields can be considered as gravitational, Newtonian, and electromagnetic, while the moments are represented by gyrostatic, perturbing, and restoring moments. Some of the perturbation methods are viewed to show how much they are involved in the solutions to this problem.

**Keywords:** Rigid Body; Rotatory Motion; Equations of Motion (EOM)

## Overview of the rigid body problem

The rotational motion of a rigid body about a fixed point in a uniform field or in a Newtonian one, is considered one of the important problems in the theoretical classical mechanics. It is well known that these problems are governed by systems of six nonlinear differential equations with three first integrals related to the kinetic energy, area, and the geometric integral [1].

The exact solutions of such systems require an urgent fourth integral. Numerous works were made to realize such integral. These attempts require operations on the results which could not be expressed explicitly in the general case except for three famous special cases; (Euler-Poinsot, Lagrange-Poisson, and Kofaleveskaya). In these cases, there are certain restrictions on the location of the body's centre of mass and on the values of the principal moments of inertia. Arkhangel'skii, Iu. A. [2] showed that this fourth algebraic integral exists only in two special cases analogous to those of Euler and Lagrange and that, other cases with single-valued integrals are not additional cases, but they can be reduced to previous cases. Other integrable cases can be found in [2-4].

Perturbation methods such as the Poincaré small parameter method (SPMP), the large parameter method (LPM), the Krylov-Bogoliubov-Mitropolski technique (KBMT), the averaging method (AM), and others [6,7] have piqued the interest of several researchers over the last five decades in obtaining the approximate solutions to this problem.

The SPMP was used to find the first terms of the series expansion of the periodic solutions of the equations of motion (EOM) of a heavy rigid body about a fixed point when the body spins rapidly about the third principal axis of the body [8-10]. It is worthy to mention that the obtained solutions contain singular points, which makes these solutions to be irregular, when the frequency of the problem takes integer values or their inverses. Treatment of emergen singularities at all has been found in [11-15] when Amer's frequency has been used. This frequency depends on the third component of the gyrostatic moment vector and on the values of the principal moments of inertia. The LPM has been used in the previous recent years to investigate the motion of the rigid body [16-18].

In [19], the KBMT was modified to solve the problem of motion of a fast spinning heavy solid about a fixed point. The influence of the gyrostatic moment vector on the rigid body motion is examined in [20-22] to avoid the appearance of singular points. The governing system of the EOM and their three integrals are reduced to another system of two second-order differential equations and one integral. The approximate solutions of this system are obtained and represented graphically to show the action of the body's parameters on the investigated motion. A general case for this problem is examined in [21], while in [22] the body's ellipsoid of inertia is considered to be close to its ellipsoid of rotation. Moreover, Euler's angels were derived to view the orientation of the body at any given instant.

The AM is frequently used for treating this problem when it is close to Lagrange's case, i.e. when the values of two principal moments are equal and different from the third one, in addition to when the center of mass is located on the dynamic symmetry axis. This method is used in [23,24] when the body is acted upon by perturbing moments in a gravitational field [25,26], when the Newtonian one is considered [25,28], when a charged body is acted upon, and [25,29] when the gyrostatic moment is acted upon. The governing system is transformed to another appropriate averaging one, in which the last system is solved analytically up to a high accuracy order.

## Conclusion

This brief study has clarified how a rigid body rotates around a fixed point when subjected to various external fields and moments. These fields have been thought of as gravitational, Newtonian, and electromagnetic, while the moments have been represented by gyrostatic, perturbing, and restoring moments. A few perturbation techniques have been reviewed to demonstrate how crucial they

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