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Research Article

# Equations and Equivalences in Physics

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## Abstract

Physics use equations without mentioning the equivalence relations on sets which produce them. For different purposes [1] they belong to factor algebras, projective scalings, topological equivalences.

Examples: $\lambda p = \lambda mv$ = h, λ wave length, p momentum, m ma ss, v speed, h Planck constant. Using Pauli spin matrices, $\sigma_1$ can stand for wave length, multiplication with the h scaled identity id matrix gives the scalar h. The (scaled) unit eigenvectors of the two span a rectangle of this area. Similarly for the angle - angular momentum Heisenberg uncertainty [17] φJ = h and E =hf for energy with matrix α and $f^{-1}$ (inverse frequency) with $\alpha^2$ space the area h. The matrix $\sigma_1$ is in the first cases substituted by scalar values. The formulas generate through noncommutativity inequalities with lower h-bounds where one member (either a coordinate or an energy) is measured unsharp in case the other is measured more precise.

**Keywords:** Equivalence; Equation; Color Charge; Measuring GF Triples; Unit Spheres

## Introduction

Equivalences in mathematics are reflexive, symmetric, transitive relations [6] and often are not directly yielding equations. In this article the issue is to point out projectivity (for instance norming a vector x to λx, λ ≠ 0) for gravity and quantum measures, using Gleason operators T [14,16]. They also add metrical changes to the usual Euclidean metrics in form of <xT,x>. How to set natural constants is discussed in relation to equations between coordinates and their associated energies. A particular emphasis is on color charges as independent octonian force of the strong interaction, using the complex, real 2-dimensional Riemannian sphere and its Moebius tranformations with color charges as their six cross ratios. Weak interaction Heegard polar decays are related to finite dihedral symmetries $D_n$, n = 1,2,3,4,6. Quantizations arise for instance through the complex residue theory. Contour integrations require that the contour about poles has to be closed. If not, such an energy cannot be stored. Topological methods extend those used in physics: Morse critical functions and higher order catastrophe parametrized sets of critical functions allow models [15,18] constructed in catastrophe theory.

## Results and Methods

A first example after a Q Higgs (or big) bang are the six color charges CC of quarks. The geometry [13] generated is a 2-dimensional ball surface $S^2$ as complex Riemannian sphere with ∞ added as north pole of $S^2$ and the geographical stereographic projection on a Gauss z-plane, z = x +iy, x,y real numbers, $i^2 = -1$ imaginary. The symmetry are the Moebius transformations MT. The MT are able to add scalars for renorming of measures, to make translations of systems by their momentum in space and to make rotations.

The equivalence relation on the MT invariant cross ratios (the six CC) is by permuting the last three members of z,0,1,∞. The number 1 can alternatively be replaced by -1. The 0,1,∞ permuted numbers present the matrices of the quark triangle symmetry $S_3$, mostly noted as dihedral $D_3$. It has α and $\sigma_1$ as members.

It is postulated that the CC are available in Q as black hole or dark matter location [10,12], but in dimension 1 as lemniscates. After the Q decay and before Planck numbers and h are set, they belong as property (a conic whirl) to 3-dimensional quarks which are brezels of genus 2. A projective duality allows to change the dimension1 to 3 in a projective (Kaluza-Klein) space 5-dimensional space $P^5$. The members of this space arise in a complex 3-dimensional space $C^3$ by setting for lines through the origin only one point of $P^5$. If the $C^3$ coordinates are enumerated by indices, the CC are attached to the coordinate 1 as red r, 2 as green g, 3 as magenta c(g) (conjugate CC), 4 as yellow c(b), 5 as turquoise c(r), 6 as b. In the same (cross ratios) equivalence class are an energy, an octonian coordinate and a symmetry for the energy GF measure: electrical force EM and length, heat, rotational energy or angular momentum, magnetic energy and time, mass, kinetic energy. Setting units for measuring the energies, they are eigenvectors of $D_3$ matrices and measure 1 in Ampere or meter, 2 in Kelvin, 3 in Joule, 4 in Tesla or second, 5 in kg, 6 in Hz.

After this setting the electromagnetic interaction EMI is set by circles U(1) or projective $P^1$ for photons, light. The universal covering of U(1) is a real line, but rolled up in time as a helix line on a cylinder. After windings counted with natural numbers f = n and rotation time T = 1/n, the equivalence relation is for the n mapped for instance to 1εU(1). In relation to the radius r (not the above CC r) of space, the Minkowski metric cone $r^2 = c^2t^2$, c speed of light, t time, describes in reduced 1-dimensional space coordinates two lines in the complex C plane with the origin 0 as common point. C in real coordinates xy is taken as a projective $P^2$ plane. In it the two lines are closed at projective infinity by two points, giving a lemniscate. If 3-dimensional z-coordinate of space rotated, the 2-dimensional double cone closes in projective infinity by a circle, giving geometrical a Horn or pitched torus. It has one transversal torus circle retraced to a singular point.

A similar pitched torus arises for the EMI cylinder which is closed at projective infinity by a singular point. Photons are set free only after a long time when neutrinos can escape from atoms and electrons in an atoms shell can jump between different spherical shells of radius r, as main quantum numbers n, emitting or absorbing photons.

Beside the two natural constants h,c the Planck numbers use k Kelvin for scaling heat towards energy and G for the gravitational constant, occuring as scalar in the Schwarzschild radius of a physical system P with mass as $Rs = 2Gm/c^2$. The MT 1/z of $S^2$ allows to invert radius r‘ inside P to radius r in the universe as $r'r = Rs^2$. A similar inversion is for speeds of dark energy inside a pinched torus v‘ to universes speeds v in $v'v = c^2$ at a Minkowski cone. It is mentioned that this metric $ds^2 = dr^2 - c^2dt^2$ has a critical Morse function for its affine geometry which in diagonal form has the matric diag[1,1,1,-1], -1 for complex imaginary time ict ($i^2 = -1$), the others for xyz-space.

For the quantum mechanical systems the measures obey the Copenhagen interpretation: From three vectors like spin s = $(s_x, s_y, s_z)$ of an orthogonal $S^2$ base triple only one can be measured in an experiment and possibly the valus for s, the other two vectors remain undetermined. The energy measuring triples GF of this kind arise in octonians, spin-like as 123 (space, measure: meter), 145 (EM, Tesla 4 or Ampere 1) , 167 (EMI, 7 rolled Kaluza Klein U(1) circle, 6 frequency, 1 wave length), 246 (heat 2), 257 (mass 5), 347 (rotation 3), 356 SI rotor of the strong interaction of functions for integrations of functions. The SI rotor is a presentation of the $D_3$ symmetry. A similar weak insteraction WI rotor exist where three wheels in the xy- or xz- or yz-plane are rotating using the third space axis. This rotor is for differentiations of functions. The measuring Gleason operators generated by these triples can have projective renormings of vectors by a common real or complex number. They also allow a probability measure on subspaces of a Hilbert space [5,8].

Concerning a Hilbert space dimension 4, the failure of commutativity for projections P, Q (in general $PQ \neq QP$, see the Heisenberg uncertainty equalities) has as consequence that an atomic lattice 4-circle has to have a central astroid [3]. As in figure 1, it is obvious that octonians are necessary for the atom coordinates. To 1,2,…,7 an octonian coordinate 0 for the CC force has to be added.

Figure 1 at left an atomic 3-cycle requires a block with 4 coordinates since the three vertices of the 3-cycle are presenting pairwise commuting coordinates, at right the outer 4-cycle has inside an astroid with 8 atoms; commuting operators on $C^4$ form a Boolean sublattice (an interval or curved line called block) and the $C^4$ lattice is the set theoretical union of its blocks; he same diagrams apply also to a real 4-dimensional Hilbert spacetime $R^4$.

In figure 2 the Fano memo for the GF triples, also coordinates are drawn as points on the intervals (or circle) and the octonian

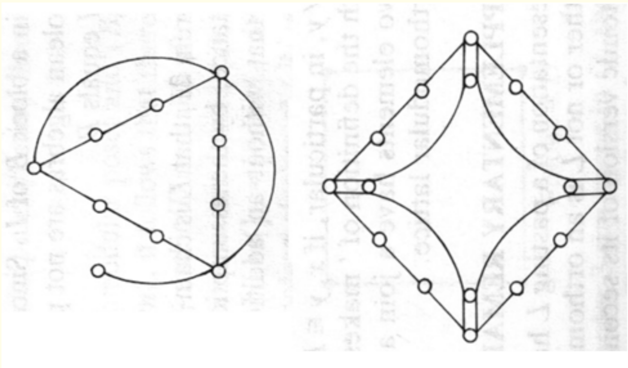

**Figure 1**

$e_0$ or 0 coordinate is set at left aside as an input vector. The octonian multiplication is different from the SI GellMann matrices and also different from a complex 4-dimensional Hilbert space $C^4$ (or real $R^4$) with inner product and the subspace lattices of figure 1. The difference is by their representants as number system, as 3x3-matrices or through a complex Hilbert space metric.

The geometries for CC, EMI, SI, WI and mass with EM are different. For CC in use is here $S^2$. For WI it is $R^4$ with the Hopf fiber bundle which maps its sphere $S^3$ down to $S^2$ and has fiber $S^1$ as a topological equivalence. This is also the fiber of the SI fiber bundle which maps the factor unit sphere $S^5$ of SI down to a complex $CP^2$ space, having as boundary $S^2$. This is the nucleon and atomic kernels octonian subspace 2356, not 1234 spacetime. For EM (with mass and frequency) a 1456 can be used.

**Figure 2:** Fano memo of the GF.

The SI geometry is a toroidal topological product of $S^3$ x $S^5$. The first three GellMann matrices present the neutral rgb-graviton whirl and its 3x3-matrices are mapped down to the WI Hopf $S^3$ Pauli matrices. They define the Hopf map, projecting $R^4$ 1234 down to space $R^3$ 123. There is a third 4-dimensional octonian subspace 1456, occuring in a Feigenbaum evolution of energies after a Higgs (big) bang. The evolution is from 0 bifurcating to 1,5, 1 bifurcating to 2,4 and 5 to 3,6. Then SI bifurcates, after that heat chaos occurs.

## Conclusions

Equivalences occur in different foms, algebraically through factor groups like $D_3$ from $S_4$ for instance. Projectivity and the GF have scalar mulitplication of vectors as equivalence. Topological equivalences are present in the fiber bundles. For equations in physics other examples can be consulted, not only the Heisenberg uncertainties equations. Subsitutions give for instance from E = hf the angular speed ω equation $E = h\omega/2\pi$. This angle is used for oscillations descriptions in $\psi = \exp(i\omega t)$, exp the exponential function of 7, t time. The first such oscillation is produced by the SI rotor (Figure 3). The blue CC vector is rotating in every of the six rotor steps. Its endpoint traces then out an oscillation as three orthogonal circles orthogonal to the quark triangle sides. It fixes the quarks barycenters (also done by the six red or green vectorial rotations).

**Figure 3:** Momentum blue vector in six rotational steps, the endpoint of this vector is rotating alternating cw clockwise and mpo counterclockwise for the oscillation

Wave equations for instance of EMI and matter waves come later on in the development of the universe. The refences for the physics related to EMI, SI, WI and their rotation or Lie groups are to articles in the internet. Below is a list of books written by the author during the time 1969 to today [2,11]. Many details are published since the Tschernobyl event. The author started then to apply her knowledge of quantum structures for modeling MINT-Wigris. 15 models which run macroscopic technical and are on her exhibition (some are in the figures below).

**Figure 4:** Models for color charges as G-compass, as polar caps on a nucleon or deuteron, a quarkgluon flow as 6 roll mill a deuteron model for 2 nucleons, construction a nucleon barycenter by the SI rotor, changing length by rgb-gravitons, a template for drawings with pencil on paper (cut out figures to roll), toroidal singular points for inverted dark radii (mass) and speeds (energy).

**Figure 5:** A SI rotor model, then a quaternions generating model, a WI model for Heegard decompositions of the Hopf sphere S3 and a model for fusion of two protons (at left drawn as two tetrahedrons with parallel quark triangles) to deuteron (with rotated triangles at right) where the dimensions of three planes is 1-dimensional projected (forming the Heisenberg inqualities).

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## Bibliography

1. Kalmbach G. Archives Kalmbach H.E. 1967-2001 (many books and Unifikate concerning the scientific and educational work of Gudrun Kalmbach H.E.) (2001).
2. Kalmbach G. ''Proceedings of the Lattice Theory Conference". Universitaet Ulm (1975).
3. Kalmbach G. "Orthomodular Lattices". Academic Press, London (1983).
4. Kalmbach HEG. Tag der Mathematik, Begleithefte Baden-Wuerttemberg, 1985-2001 (2001).
5. Kalmbach G. "Measures and Hilbert Lattices". World Scientific Publishers, Singapore (1986).
6. Kalmbach G. Diskrete Mathematik, Vieweg, Wiesbaden (1988).
7. Kalmbach HEG. "Talent Development in Mathematics". Science and Technology I, II, Aegis-Verlag, Ulm, 1989/1990.
8. Kalmbach HEG. "Quantum Structures 1,2, in". *International Journal of Theoretical Physics* 1991 (1992).
9. Kalmbach G. Fragebogen zur Frauenfoerderung in Mathematik, 1991-1994, Universitaet Ulm, with articles, evaluation and discussions, in: MINT 39 (1994).
10. Kalmbach G. (with Martin Grimm editions.), Skripte, for local MINT Courses, Universitaet Ulm, 1993-1999.
11. Kalmbach G. Mathematik - bunt gemischt 1,2, (2 with R. Schweizer), Becker Verlag, Velten (1996).
12. Kalmbach G. MINT (Mathematik, Informatik, Naturwissenschaften, Technik), Aegis-Verlag Ulm and MINT Verlag Bad Woerishofen 1-60 1997-2021.
13. Kalmbach G Geometrie. Gruppen und Logik, Universitaet Ulm, Lecture Notes (1998): 1-126.
14. Kalmbach G. Quantum Measures and Spaces, Kluwer, Dordrecht (1998).
15. G Kalmbach. Animation (programmer: S. Knupfer), internet open access 2000-2010.
16. G Kalmbach. Quantum Mathematics, RGN Publishers, Delhi (2015).
17. G Kalmbach. MINT-Wigris, Scholars' Press, Beau Bassin (2017).
18. G Kalmbach. H.E. with U. Eberspaecher, MINT-Wigris Tool Bag and handbook (page numbers belowfor the content), Bad Woerishofen,, contents: MINT-Wigris E-Tools Models are and Color charges of Quarks as complex Force 1, 2 6 roll mill 3, Quark 5,SI rotor and blue vectors oscillation 7,Gravity and its stretching-squeezing 9,Fusion 15,Spin/magnetic momentum positions and the neutral leptons case 17, Electromagnetism 20,Electromagnetic Interaction, Lissajous and Dihedrals 23,Heegard Decompositions 26,Energy Exchange 28,Quanta Measuring 30,Dark Universe 3, Symmetry Breaking and Catastrophes 35 (2019).