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Mini Review

A Dual Approach for Model Construction of Two-Dimensional Horizontal Flow

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Abstract

The two-dimensional horizontal flow model in the classical integration approach is integrated from the three-dimensional Navier-Stokes system of equations. Using the classical theory, the integral is taken directly from the bed to the free water surfaces. Consequently, the effects between the channel bed and free water surface, in the process of integration, was disappeared. However, with the proposed dual-process approach, the integral can be performed locally several times. The receiving equations thus allow to contain many physical phenomena which may be lost in the classical integral process. As a result, the derived model based on the proposed dual approach will be more complex and accurate than the classical one. In this paper, the authors perform twice integrals. The improved two-dimensional horizontal flow model was received from the dual approach which allows the calculation of flow parameters, which, having the unusual phenomena in the channel as solid objects, liquids containing other added ingredients, external forces, reversals, and so on. Moreover, it provides flexible parameter adjustment based on the experimental data.

Keywords: Dual Approach; Classical Integration Approach; Average of the Integrals; Two-Dimensional Horizontal Flow (2DH); Shallow Water Equation (SWE)

Introduction

Flows are three-dimensional, but there are cases where they can be considered as horizontal flow (such as shore overflows, bay currents, estuary, coastal flows). There are flows seen as one-dimension (flow of water in canals, small and medium sized rivers). The flow in rivers are described by the three-dimensional Navier-Stokes system of equations [3,4,7,9-12]. However, it is difficult to obtain the direct solution of the three-dimensional system of equations due to many mathematical difficulties in mathematics, computational time and data collection for verification [3-6,8,12]. Therefore, in practice, the mathematical models are often much simpler in order to simplify the real problems in many cases [5,9,11]; One of the simplifications is to only integrate along the vertical direction of the three-dimensional system Navier-Stokes of equations, which yields a horizontal two-dimensional flow equation [3,7,12].

Depending on the average of the integrals, as well as the boundary conditions, assigned to the averages, we obtain two-dimensional models. With the horizontal flow model, the vertical velocity is ignored; the horizontal velocities u and v are taken as the mean of the flow depths. On the other hand, there are many different numerical methods with different precision, different algorithmic complexity, which can be used for the computation.

At present, in the world there are many studies on establishing the governing equation of two-dimensional flow mathematical models, namely the classical average theory; However, in certain cases this two-dimensional model cannot be satisfied, for example when the flow has abnormal phenomena in the conduit such as solid objects, liquid containing other components, external body force f_x, f_y, f_z, changing direction of flow.

In this paper, the authors set up a two-dimensional horizontal model in a dual approach [1,2].

According to the dual approach taken in this paper, the integration process is divided into two phases. First, the integral is taken directly from the bed to the intermediate surface between the bed and the water surface. Then, integrating the second time from the bed to the water surface, we will receive differential equations in a dual approach [1,2]. This approach would be more complicated than the canonical method, but in return we will obtain the lost effect in the classical method. Moreover, it provides flexible parameter adjustment based on the experimental data.

A dual approach to construct two-dimensional horizontal equations (2DH)

The three-dimensional (3D) Navier-Stokes system of equations have been approximated by Reynolds as follows.

The continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

The equations of momentum in x-, y- and z- directions:

$$\frac{\partial u}{\partial t}+\frac{\partial(u.u)}{\partial x}+\frac{\partial(u.v)}{\partial y}+\frac{\partial(u.\mathrm{w})}{\partial z}= \frac{1}{\rho}.F_x-\frac{1}{\rho}.\frac{\partial p}{\partial x}+E_{xx}\frac{\partial^2 u}{\partial x^2}+E_{xy}\frac{\partial^2 u}{\partial y^2}+E_{xz}\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{\partial(u.v)}{\partial x}+\frac{\partial(v.v)}{\partial y}+\frac{\partial(v.\mathrm{w})}{\partial z}= \frac{1}{\rho}.F_y-\frac{1}{\rho}.\frac{\partial p}{\partial y}+E_{yx}\frac{\partial^2 v}{\partial x^2}+E_{yy}\frac{\partial^2 v}{\partial y^2}+E_{yz}\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial \mathrm{w}}{\partial t}+\frac{\partial(u.\mathrm{w})}{\partial x}+\frac{\partial(v.\mathrm{w})}{\partial y}+\frac{\partial(\mathrm{w.w})}{\partial z}= \frac{1}{\rho}.F_z-\frac{1}{\rho}.\frac{\partial p}{\partial z}+E_{zx}\frac{\partial^2 \mathrm{w}}{\partial x^2}+E_{zy}\frac{\partial^2 \mathrm{w}}{\partial y^2}+E_{zz}\frac{\partial^2 \mathrm{w}}{\partial z^2} \tag{4}$$

Where: u, v, w are correspondent velocity components of x, y and z axes; F_x, F_y, F_z are Coriolis forces; p is the pressure; E_{xx}, E_{yy}, E_{zz}, E_{xz}, E_{zx}, E_{yx}, E_{xy}, E_{yz}, E_{zy} are the eddy viscosity coefficients; g is acceleration of gravity.

Under certain conditions, 2DH can be described by integrating equations (1), (2), (3), (4) by the dual approach.

The conditions are:

- Uncompressed fluid;
- Vertical velocity is negligible and much smaller than horizontal velocity components u and v (u >> w, v >> w).

Under the conditions above, the Reynolds equation (4) is reduced to $\partial p/\partial z=-\rho g$.

The vertical pressure distribution is hydrostatic and the datum is chosen for the position z = 0; on the free water surface B (interaction of gas and water) z = η, and at the bed of river A (interactions between sediment and water) z = -h (Figure 1).

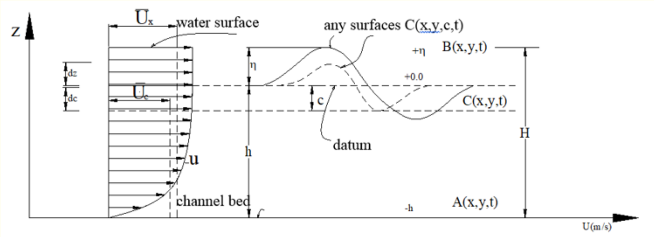

Figure 1: Sketch describing velocity profile in the x direction.

There are two different mean values and relations between the mean values in the integration process:

- The mean value in terms of x and y from bed A to any point C, between the bed and free water surface, is $\overline{U}_c$, $\overline{V}_c$.
- The mean value in terms of x and y from bed A to free water surface B is $\overline{U}_x$, $\overline{V}_y$.
- The mean values in terms of x and y are different according to the chosen of the surface to be calculated. There are relations of velocity between any surface C with the bed A and the average velocity between the surface B and the bed of the river A.

Average velocity $\overline{U}_x$, $\overline{V}_y$ in depth from river bed A(x, y, t) to free water surface B(x,y,t) with x and y direction correspondent:

$$\overline{U}_x=\frac{1}{h+\eta}\int_{-h}^{\eta}u dz; \qquad \overline{V}_y=\frac{1}{h+\eta}\int_{-h}^{\eta}v dz;$$
$$\int_{-h}^{\eta}u dz=\overline{U}_x.(h+\eta); \int_{-h}^{\eta}v dz=\overline{V}_y.(h+\eta) \tag{5}$$

Total height of water column from bed A to free water surface B is H = h + η.

The average velocity $\overline{U}_c$, $\overline{V}_c$, in terms of the depth x, y from the datum A(x, y, t) to any intermediary surface C(x, y, t):

$$\overline{U}_c=\frac{1}{h+c}\int_{-h}^{c}u.dz; \qquad \overline{V}_c=\frac{1}{h+c}\int_{-h}^{c}v.dz;$$
$$\int_{-h}^{c}u.dz=\overline{U}_c(h+c); \int_{-h}^{c}v.dz=\overline{V}_c(h+c) \tag{6}$$

Other mean values $\overline{U_cU}_c$, $\overline{U_cV}_c$, $\overline{V_cV}_c$ is defined as follows:

$$\overline{U_cU}_c=\frac{1}{h+c}\int_{-h}^{c}u.u.dz;\quad \overline{V_cV}_c=\frac{1}{h+c}\int_{-h}^{c}v.v.dz;$$
$$\overline{U_cV}_c=\frac{1}{h+c}\int_{-h}^{c}u.v.dz;\quad \int_{-h}^{c}u.v.dz=\overline{U_cV}_c(h+c) \tag{7}$$

Total height of water column from bed A to water surface C is $H_c = h + c$.

Boundary conditions of the problem:

Condition in any surfaces C(x, y ,c, t) at $Z_c = c$ located between the bed A and the free water surface B:

$$-\left[u.\frac{\partial c}{\partial x}+v.\frac{\partial c}{\partial y}-\mathrm{w}\right]_{Z=Z_c=c}=-\left[\bar{U}_c.\frac{\partial c}{\partial x}+\bar{V}_c.\frac{\partial c}{\partial y}-\mathrm{w}\right]=\frac{\partial c}{\partial t} \tag{8}$$

Boundary conditions at free water surfaces B(x,y,z,t):

$$-\left[u.\frac{\partial \eta}{\partial x}+v.\frac{\partial \eta}{\partial y}-\mathrm{w}\right]_{Z=Z_s=\eta}=\left[\alpha_1\bar{U}_c.\frac{\partial \eta}{\partial x}+\beta_1\bar{V}_c.\frac{\partial \eta}{\partial y}-\mathrm{w}\right]=\frac{\partial \eta}{\partial t} \tag{9}$$

Boundary conditions at channel bed A(x, y, z, t):

$$\left[u.\frac{\partial(-h)}{\partial x}+v.\frac{\partial(-h)}{\partial y}-\mathrm{w}\right]_{Z=Zb=-h}=0 \tag{10}$$

Integrating the continuity equation based on the dual approach

Local integration

Integrating the 3D continuity equation from bed A to any intermediate surface C between bed A and free water surface B (see figure 1):

$$\frac{1}{c+h}\int_{Z_b}^{Z_c}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \mathrm{w}}{\partial z}\right).dz=\frac{1}{c+h}\int_{-h}^{c}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \mathrm{w}}{\partial z}\right).dz=0$$

- Multiplying the equation with (c + h) and we get

The final integral:

$$\int_{-h}^{c}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \mathrm{w}}{\partial z}\right).dz=\int_{-h(x,y,t)}^{c(x,y,t)}\frac{\partial u}{\partial x}.dz+\int_{-h(x,y,t)}^{c(x,y,t)}\frac{\partial v}{\partial y}.dz+\int_{-h(x,y,t)}^{c(x,y,t)}\frac{\partial \mathrm{w}}{\partial z}.dz=0\ \int_{-h(x,y,t)}^{c(x,y,t)}\frac{\partial u}{\partial x}.dz+\int_{-h(x,y,t)}^{c(x,y,t)}\frac{\partial v}{\partial x}.dz+\mathrm{w}(c)-\mathrm{w}(-h)=0$$

Applying the Leibnitz principle and combine the upper and lower boundary conditions of equations (8) and (10), we obtain:

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{c}u.dz+\frac{\partial}{\partial y}\int_{-h}^{c}v.dz=0$$

$$\frac{\partial c}{\partial t}+\frac{\partial\left[\bar{U}_c.(c+h)\right]}{\partial x}+\frac{\partial\left[\bar{V}_c.(c+h)\right]}{\partial y}=0$$

As a consequence of the transformation, we obtain this continuity equation as follows:

$$\frac{\partial(c+h)}{\partial t}+\frac{\partial\left[\bar{U}_c.(c+h)\right]}{\partial x}+\frac{\partial\left[\bar{V}_c.(c+h)\right]}{\partial y}=0 \tag{11}$$

Or

$$\frac{\partial c}{\partial t}+\frac{\partial\left[\bar{U}_c.(c+h)\right]}{\partial x}+\frac{\partial\left[\bar{V}_c.(c+h)\right]}{\partial y}=0 \tag{12}$$

Global integration

Integrating again equation (12) from bed A to water surface B (See figure 1):

$$\int_{Z_b}^{Z_s}\left(\frac{\partial c}{\partial t}+\frac{\partial\left[\bar{U}_c.(c+h)\right]}{\partial x}+\frac{\partial\left[\bar{V}_c.(c+h)\right]}{\partial y}\right)dc=$$
$$\int_{-h}^{\eta}\left(\frac{\partial c}{\partial t}+\frac{\partial\left[\bar{U}_c.(c+h)\right]}{\partial x}+\frac{\partial\left[\bar{V}_c.(c+h)\right]}{\partial y}\right)dc=0 \tag{13}$$

After many transformations, we receive the continuity equation as follows:

$$\frac{\partial H^2}{\partial t}+\frac{\partial\left(\alpha_1\bar{U}_cH^2\right)}{\partial x}+\frac{\partial\left(\beta_1\bar{V}_cH^2\right)}{\partial y}=0 \tag{14}$$

In which, H is the height of the water column from the bed A to the free water surface B, which $\bar{U}_c$ is the average velocity from the bed to any surface C; similarly $\bar{V}_c$ is from bed A to any intermediate surface C and $\bar{V}_x$ from bed A to free water surface B. The coefficients α_1 and β_1 are definition as follow

$$\alpha_1=\frac{\bar{U}_x}{\bar{U}_c}=\frac{2(Z_c-Z_b)}{(Z_s-Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c}u.dz\right)dc}{\int_{Z_b}^{Z_c}u.dz}=\frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c}u.dz\right)dc}{\int_{-h}^{c}u.dz}$$

$$\beta_1 = \frac{\bar{V}_y}{\bar{V}_c} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} v.dz\right)dc}{\int_{Z_b}^{Z_c} v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} v.dz\right)dc}{\int_{-h}^{c} v.dz}$$

Integrating the momentum equation based on the dual approach in the x direction

Local integration

Integrating the momentum equation from bed A to any surface C between the channel bed A and free water surface B.

Based on the assumptions for condition of the shallow water equation, we get $\partial p / \partial z = -\rho.g$, and integrating from any surface C(Z_c) to elevation surface Z.

$$\int_{p_c(x,y)}^{p(x,y,t)} dp = -\int_{Z_c=c}^{Z} \rho.g.dz = -\rho.g.\int_{c}^{z} dz = \rho.g.(c-z)$$

$$p - p_c = -\int_{Z_c=c}^{Z} \rho.g.dz = -\rho.g.\int_{c}^{Z} dz = \rho.g.(c-z)$$

$$p = \rho.g.(c-z) + p_c = \rho.g.(c-z) + \rho.g.(\eta - c) = \rho.g.(\eta - z)$$

In which, the pressure at any surface C(x, y, c, t) get in between the free surface B and the channel bed A:

$$p_c = \rho.g.(\eta - c)$$
$$-\frac{\partial p}{\partial x} = -\rho.g.\frac{\partial \eta}{\partial x} + \rho.g.\frac{\partial z}{\partial x} \quad (15)$$

Substituting equation (15) into (2) after integrating from bed A to any surface C, we obtain:

$$\underbrace{\int_{-h}^{c}\frac{\partial u}{\partial t}.dz}_{Ax1} + \underbrace{\int_{-h}^{c}\frac{\partial(u.u)}{\partial x}.dz}_{Bx2} + \underbrace{\int_{-h}^{c}\frac{\partial(u.v)}{\partial y}.dz}_{Cx3} + \underbrace{\int_{-h}^{c}\frac{\partial(u.w)}{\partial z}.dz}_{Dx4} =$$
$$\underbrace{\int_{-h}^{c}\frac{1}{\rho}.F_x.dz}_{Ex5} - \underbrace{\int_{-h}^{c} g.\frac{\partial(\eta - z)}{\partial x}dz}_{Fx6} \quad (16)$$
$$+ \underbrace{\int_{-h}^{c}\left(E_{xx}\frac{\partial^2 u}{\partial x^2} + E_{xy}\frac{\partial^2 u}{\partial y^2}\right).dz}_{Gx7} + \underbrace{\int_{-h}^{c} E_{xz}\frac{\partial^2 u}{\partial z^2}.dz}_{Hx8}$$

Using the Leibniz rule and the definite integrals from the bed to the surface C between the bed A and the free water surface B, and combining boundary conditions, we obtain the final form of the momentum equation x as follows:

$$\frac{\partial}{\partial t}\left[\bar{U}_c(c+h)\right] + \frac{\partial}{\partial x}\left[\overline{U_c.U_c}.(c+h)\right] + \frac{\partial}{\partial y}\left[\overline{U_c.V_c}(c+h)\right] = -g.(c+h)\frac{\partial \eta}{\partial x}$$
$$+\left(E_{xx}^c\frac{\partial^2 \bar{U}_c}{\partial x^2} + E_{xy}^c\frac{\partial^2 \bar{U}_c}{\partial y^2}\right).(c+h) + \frac{\tau_{zx}(c)}{\rho} - \frac{\tau_{zx}(-h)}{\rho} + 2.(c+h).\varpi.\bar{V}_c.\sin\theta \quad (17)$$

Global integration

Integrating again equation (17) from bed A to free water surface B (See figure 1):

$$\frac{\partial}{\partial t}\left[\alpha_1\bar{U}_c.H^2\right] + \frac{\partial}{\partial x}\left[\alpha_2\overline{U_c.U_c}.H^2\right] + \frac{\partial}{\partial y}\left[\alpha_3\overline{U_c.V_c}.H^2\right] = -gH^2\frac{\partial \eta}{\partial x}$$
$$+\left(\alpha_4 E_{xx}^x\frac{\partial^2 \bar{U}_c}{\partial x^2} + \alpha_5 E_{xy}^x\frac{\partial^2 \bar{U}_c}{\partial y^2}\right)H^2 + \frac{2H.\tau_{zx}(\eta)}{\rho} - \frac{2H.\tau_{zx}(-h)}{\rho} + 2.\varpi.\beta_1\bar{V}_c.H^2.\sin\theta;$$

Where: $\tau_{zx}(\eta)$ is the wind shear stress on the water surface, $\tau_{zx}(\eta)=\rho_a.C^*.W_x.W$; where ρ_a is the specific mass of the air; C* is the Ekman coefficient; W_x is the wind velocity on the water surface in the x-direction; W_y is the wind velocity in y direction. The principle wind velocity $W=(W_x^2+W_y^2)^{0.5}$ and τ(-h) is the shear stress at the bed, which is due to friction at the bed, according to Chezy $\tau_{zx}(-h) = \rho.g\left|\bar{U}\right|\bar{U}/C^2 = \tau_{bx}$.

Substituting and transformation the above equation becomes:

$$\frac{\partial}{\partial t}\left[\alpha_1\bar{U}_c.H^2\right] + \frac{\partial}{\partial x}\left[\alpha_2\overline{U_c.U_c}.H^2\right] + \frac{\partial}{\partial y}\left[\alpha_3\overline{U_c.V_c}.H^2\right] = -gH^2\frac{\partial \eta}{\partial x}$$
$$+\left(\alpha_4 E_{xx}^c\frac{\partial^2 \bar{U}_c}{\partial x^2} + \alpha_5 E_{xy}^c\frac{\partial^2 \bar{U}_c}{\partial y^2}\right)H^2 + \frac{2\rho_a}{\rho}C^*.W_x.W.H \quad (18)$$
$$-2g.\alpha_1\overline{U_c}.n^2\sqrt{\alpha_1^2\overline{U_c}^2 + \beta_1^2\overline{V_c}^2}\,/\,H^{-2/3} + 2\varpi.\beta_1.\bar{V}_c.H^2.\sin\theta;$$

In which, $\bar{U}_c$, $\bar{V}_c$ is the average velocity component at any surface C(x,y,t) in the direction x, y; ϖ is the magnitude of the earth angular velocity; θ is the latitude degree of the water body of interest; H = η + h; η is the water level compared to the datum surface; h is the depth from the datum surface to the river bed; E_{xx}^c, E_{xy}^c is eddy viscosity coefficient; ρ is the specific gravity of the water; α_1, β_1, α_2, α_3, α_4, α_5 are coefficients which define as follows :

$$\alpha_1 = \frac{\bar{U}_x}{\bar{U}_c} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} u.dz\right)dc}{\int_{Z_b}^{Z_c} u.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} u.dz\right)dc}{\int_{-h}^{c} u.dz}.$$

$$\beta_1 = \frac{\bar{V}_y}{\bar{V}_c} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} v.dz\right)dc}{\int_{Z_b}^{Z_c} v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} v.dz\right)dc}{\int_{-h}^{c} v.dz}$$

$$\alpha_2 = \frac{\overline{U_x.U_x}}{\overline{U_c}.\overline{U_c}} = \frac{2(Z_c - Z_b)}{(Z_s-Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} u.u.dz\right)dc}{\int_{Z_b}^{Z_c} u.u.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c}u.u.dz\right)dc}{\int_{-h}^{c}u.u.dz}$$

$$\alpha_3 = \frac{\overline{U_x.V_y}}{\overline{U_c}.\overline{V_c}} = \frac{2(Z_c - Z_b)}{(Z_s-Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} u.v.dz\right)dc}{\int_{Z_b}^{Z_c} u.v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c}u.v.dz\right)dc}{\int_{-h}^{c}u.v.dz}$$

$$\alpha_4 = \frac{E^x_{xx}\frac{\partial^2 \overline{U_x}}{\partial x^2}}{E^c_{xx}\frac{\partial^2 \overline{U_c}}{\partial x^2}} = \frac{2(Z_s - Z_b)}{(Z_s-Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(E^c_{xx}\int_{Z_b}^{Z_c} \frac{\partial^2 u}{\partial x^2}.dz\right)dc}{E^c_{xx}\int_{Z_b}^{Z_c} \frac{\partial^2 u}{\partial x^2}.dz} = \frac{2(h+c)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(E^c_{xx}\int_{-h}^{c}\frac{\partial^2 u}{\partial x^2}.dz\right)dc}{E^c_{xx}\int_{-h}^{c}\frac{\partial^2 u}{\partial x^2}.dz}$$

$$\alpha_5 = \frac{E^x_{xx}\frac{\partial^2 \overline{U_x}}{\partial y^2}}{E^c_{xx}\frac{\partial^2 \overline{U_c}}{\partial y^2}} = \frac{2(Z_s - Z_b)}{(Z_s-Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(E^c_{xx}\int_{Z_b}^{Z_c} \frac{\partial^2 u}{\partial y^2}.dz\right)dc}{E^c_{xx}\int_{Z_b}^{Z_c} \frac{\partial^2 u}{\partial y^2}.dz} = \frac{2(h+c)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(E^c_{xx}\int_{-h}^{c}\frac{\partial^2 u}{\partial y^2}.dz\right)dc}{E^c_{xx}\int_{-h}^{c}\frac{\partial^2 u}{\partial y^2}.dz}$$

Integrating the momentum equation based on the dual approach in the y direction

Local integration

Integrating the momentum equation from bed A to any surface C between the channel bed A and free water surface B.

The same with equation (15) of section 2.2, we have:

$$-\frac{\partial p}{\partial y} = -\rho.g.\frac{\partial \eta}{\partial y} + \rho.g.\frac{\partial z}{\partial y} \quad (19)$$

Substituting equation (19) into (3) and integrating from bed A to any surface C, we obtain.

$$\underbrace{\int_{-h}^{c}\frac{\partial v}{\partial t}.dz}_{Ay1} + \underbrace{\int_{-h}^{c}\frac{\partial(u.v)}{\partial x}.dz}_{By2} + \underbrace{\int_{-h}^{c}\frac{\partial(v.v)}{\partial y}.dz}_{Cy3} + \underbrace{\int_{-h}^{c}\frac{\partial(v.\mathrm{w})}{\partial z}.dz}_{Dy4} =$$
$$\underbrace{\int_{-h}^{c}\frac{1}{\rho}.F_y.dz}_{Ey5} - \underbrace{\int_{-h}^{c} g.\frac{\partial(\eta - z)}{\partial y}dz}_{Fy6} \quad (20)$$
$$+\underbrace{\int_{-h}^{c}\left(E_{yx}\frac{\partial^2 v}{\partial x^2} + E_{xy}\frac{\partial^2 v}{\partial y^2}\right).dz}_{Gy7} + \underbrace{\int_{-h}^{c}E_{yz}\frac{\partial^2 v}{\partial z^2}.dz}_{Hy8}$$

Using the Leibniz rule and the definite integrals from the bed to any surface C between the bed A and the free water surface B, and combining boundary conditions, we obtain the final form of the momentum equation for y direction as follows:

$$\frac{\partial}{\partial t}\left[\overline{V}_c(c+h)\right] + \frac{\partial}{\partial x}\left[\overline{U_c.V_c}.(c+h)\right] + \frac{\partial}{\partial y}\left[\overline{V_c.V_c}(c+h)\right] = -g.(c+h)\frac{\partial \eta}{\partial y}$$
$$+\left(E^c_{yx}\frac{\partial^2 \overline{V}_c}{\partial x^2} + E^c_{xy}\frac{\partial^2 \overline{V}_c}{\partial y^2}\right).(c+h) + \frac{\tau_{zy}(c)}{\rho} - \frac{\tau_{zy}(-h)}{\rho} + 2.(c+h).\varpi.\overline{U}_c.\sin\theta \quad (21)$$

Global integration

Integrating again equation (21) from bed A to free water surface B (see figure 1):

$$\underbrace{\int_{-h}^{\eta}\frac{\partial}{\partial t}\overline{V}_c(c+h)dc}_{DN.y1} + \underbrace{\int_{-h}^{\eta}\frac{\partial}{\partial x}\overline{U_c.V_c}.(c+h)dc}_{DN.y2} + \underbrace{\int_{-h}^{\eta}\frac{\partial}{\partial y}\overline{V_c.V_c}.(c+h)dc}_{DN.y3} =$$
$$\underbrace{\int_{-h}^{\eta}\left[-g.(c+h)\frac{\partial\eta}{\partial y}\right]dc}_{DN.y4} + \underbrace{\int_{-h}^{\eta}\left[(c+h).\left(E^c_{xx}\frac{\partial^2\overline{U}_c}{\partial x^2} + E^c_{xy}\frac{\partial^2\overline{U}_c}{\partial y^2}\right)\right]dc}_{DN.y5} + \quad (22)$$
$$+\underbrace{\int_{-h}^{\eta}\frac{\tau_{zy}(c)}{\rho}dc}_{DN.y6} - \underbrace{\int_{-h}^{\eta}\frac{\tau_{zy}(-h)}{\rho}dc}_{DN.y7} + \underbrace{\int_{-h}^{\eta}2.(c+h).\varpi.\overline{U}_c.\sin\theta.dc}_{DN.y8}$$

The equation (22) is transformated as follows:

$$\frac{\partial}{\partial t}\left[\beta_1\overline{V}_c.H^2\right] + \frac{\partial}{\partial x}\left[\beta_2\overline{U_c.V_c}.H^2\right] + \frac{\partial}{\partial y}\left[\beta_3\overline{V_c.V_c}.H^2\right] = -gH^2\frac{\partial\eta}{\partial y}$$
$$+\left(\beta_4 E^y_{yx}\frac{\partial^2\overline{V}_c}{\partial x^2} + \beta_5 E^y_{xy}\frac{\partial^2\overline{V}_c}{\partial y^2}\right)H^2 + \frac{2.H.\tau_{zy}(\eta)}{\rho} - \frac{2.H.\tau_{zy}(-\mathrm{h})}{\rho} + 2.\varpi.\alpha_1.\overline{U}_c.\mathrm{H}^2.\sin\theta;$$

Where: $\tau_{zy}(\eta)$ is the wind shear stress on the water surface; $\tau_{zy}(\eta)=\rho_a.C^*.W_y.W$ where ρ_a is the specific mass of the air; C* is the Ekman coefficient; W_y is the wind velocity on the water surface in the y-direction; W_x is the wind velocity in the x direction. The principle wind velocity $W=(W_x^2+W_y^2)^{0.5}$ and $\tau(-h)$ is the shear stress the at the bed , which is due to friction at the bed , according to Chezy coefficient: $\tau_{zy}(-h) = \rho.g\left|\overline{V}\right|\overline{V}/C^2 = \tau_{by}$.

Substituting and transformation the above equation becomes:

$$\frac{\partial}{\partial t}\left[\beta_1\overline{V}_c.H^2\right] + \frac{\partial}{\partial x}\left[\beta_2\overline{U_c.V_c}.H^2\right] + \frac{\partial}{\partial y}\left[\beta_3\overline{V_c.V_c}.H^2\right] = -gH^2\frac{\partial\eta}{\partial y}$$
$$+\left(\beta_4 E^c_{yx}\frac{\partial^2\overline{V}_c}{\partial x^2} + \beta_5 E^c_{xy}\frac{\partial^2\overline{U}_c}{\partial y^2}\right)H^2 + \frac{2\rho_a}{\rho}C^*.W_y.W.H$$
$$-2g.\beta_1\overline{V}_c.n^2\sqrt{\alpha_1^2\overline{U}_c^{\,2} + \beta_1^2\overline{V}_c^{\,2}}\,/\,H^{-2/3} + 2\varpi.\alpha_1.\overline{U}_c.\mathrm{H}^2.\sin\theta;$$

In which, $\overline{U}_c$, $\overline{V}_c$ is the average velocity component at any surface C (x, y, t) in the correspondent direction x, y; ϖ is the magnitude of the earth angular velocity; θ is the latitude degree of the water body of interest; H = η + h; E^c_{xx}, E^c_{xy} is eddy viscosity coefficient; ρ is the specific gravity of the water; α_1, β_1, β_2, β_3, β_4, β_5 are coefficients which define as follows:

$$\beta_1 = \frac{\overline{V}_y}{\overline{V}_c} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} v.dz\right)dc}{\int_{Z_b}^{Z_c} v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} v.dz\right)dc}{\int_{-h}^{c} v.dz}$$

$$\beta_2 = \frac{\overline{U_x.V_y}}{\overline{U_c.V_c}} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} u.v.dz\right)dc}{\int_{Z_b}^{Z_c} u.v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} u.v.dz\right)dc}{\int_{-h}^{c} u.v.dz}$$

$$\beta_3 = \frac{\overline{V_y.V_y}}{\overline{V_c.V_c}} = \frac{2(Z_c - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(\int_{Z_b}^{Z_c} u.v.dz\right)dc}{\int_{Z_b}^{Z_c} u.v.dz} = \frac{2(c+h)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(\int_{-h}^{c} u.v.dz\right)dc}{\int_{-h}^{c} u.v.dz}$$

$$\beta_4 = \frac{E^y_{yx}\frac{\partial^2 \overline{V}_y}{\partial x^2}}{E^c_{yx}\frac{\partial^2 \overline{V}_c}{\partial x^2}} = \frac{2(Z_s - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(E^c_{yx}\int_{Z_b}^{Z_c}\frac{\partial^2 v}{\partial x^2}dz\right)dc}{E^c_{yx}\int_{Z_b}^{Z_c}\frac{\partial^2 v}{\partial x^2}.dz} = \frac{2(h+c)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(E^c_{yx}\int_{-h}^{c}\frac{\partial^2 v}{\partial x^2}.dz\right)dc}{E^c_{yx}\int_{-h}^{c}\frac{\partial^2 v}{\partial x^2}.dz}$$

$$\beta_5 = \frac{E^y_{yy}\frac{\partial^2 \overline{V}_y}{\partial y^2}}{E^c_{yy}\frac{\partial^2 \overline{U}_c}{\partial y^2}} = \frac{2(Z_s - Z_b)}{(Z_s - Z_b)^2}\frac{\int_{Z_b}^{Z_s}\left(E^c_{yy}\int_{Z_b}^{Z_c}\frac{\partial^2 v}{\partial y^2}dz\right)dc}{E^c_{yy}\int_{Z_b}^{Z_c}\frac{\partial^2 v}{\partial y^2}.dz} = \frac{2(h+c)}{(\eta+h)^2}\frac{\int_{-h}^{\eta}\left(E^c_{yy}\int_{-h}^{c}\frac{\partial^2 v}{\partial y^2}.dz\right)dc}{E^c_{xx}\int_{-h}^{c}\frac{\partial^2 v}{\partial y^2}.dz}$$

The equation system describes the two-dimensional horizontal flow mathematical model of the open channel in the dual approach

Rewriting the above set of equations (14), (18), (23), we receive the two-dimensional horizontal flow model in the dual approach which are called the improved shallow-water equations (ISWE) as follows:

$$\frac{\partial H^2}{\partial t} + \frac{\partial}{\partial x}\left[\alpha_1 \overline{U}_c H^2\right] + \frac{\partial}{\partial y}\left[\beta_1 \overline{V}_c H^2\right] = 0 \qquad (24)$$

$$\frac{\partial}{\partial t}\left[\alpha_1 \overline{U}_c.H^2\right] + \frac{\partial}{\partial x}\left[\alpha_2 \overline{U_c.U_c}.H^2\right] + \frac{\partial}{\partial y}\left[\alpha_3 \overline{U_c.V_c}.H^2\right] = -gH^2\frac{\partial \eta}{\partial x} + \left(\alpha_4 E^c_{xx}\frac{\partial^2 \overline{U}_c}{\partial x^2} + \alpha_5 E^c_{xy}\frac{\partial^2 \overline{U}_c}{\partial y^2}\right)H^2 + \frac{2\rho_a}{\rho}C^*.W_x.W.H - 2g.\alpha_1\overline{U}_c.n^2\sqrt{\alpha_1^2\overline{U}_c^2 + \beta_1^2\overline{V}_c^2}\,/\,H^{-2/3} + 2\varpi.\beta_1.\overline{V}_c.H^2.\sin\theta; \qquad (25)$$

$$\frac{\partial}{\partial t}\left[\beta_1 \overline{V}_c.H^2\right] + \frac{\partial}{\partial x}\left[\beta_2 \overline{U_c.V_c}.H^2\right] + \frac{\partial}{\partial y}\left[\beta_3 \overline{V_c.V_c}.H^2\right] = -gH^2\frac{\partial \eta}{\partial y} + \left(\beta_4 E^c_{yx}\frac{\partial^2 \overline{V}_c}{\partial x^2} + \beta_5 E^c_{yy}\frac{\partial^2 \overline{U}_c}{\partial y^2}\right)H^2 + \frac{2\rho_a}{\rho}C^*.W_y.W.H - 2g.\beta_1\overline{V}_c.n^2\sqrt{\alpha_1^2\overline{U}_c^2 + \beta_1^2\overline{V}_c^2}\,/\,H^{-2/3} + 2\varpi.\alpha_1.\overline{U}_c.H^2.\sin\theta; \qquad (26)$$

In which $\overline{U}_c$, $\overline{V}_c$ are the average velocity component from the bed to any surface C(x,y,t), correspondent the x and y direction; $\overline{U}_x$, $\overline{V}_y$ are the average velocity components from the bed to the water surface in x and y direction; E^c_{xx}, E^c_{xy}, E^c_{yx}, E^c_{yy} are eddy viscosity coefficients; ρ is the specific gravity of the water; α_1, β_1, α_2, α_3, α_4, α_5, β_1, β_2, β_3, β_4, β_5 are the correct coefficients; C* is the drag coefficient of wind; W_x, W_y are the components of wind velocity in x and y directions respectively; W is the principle wind velocity; ϖ is the magnitude of the earth angular velocity; θ is the latitude degree of the water body of interest; η is the water level compared to the datum.

Comment: The system of equations (24), (25), (26) are more general than the classic 2DH flow equations system; It is easily to see that the classic 2DH flow system of equations is a special case of the general form equations system.

In fact, if $\frac{\partial AH^2}{\partial t_i} \approx 2H.\frac{\partial AH}{\partial t_i}$; $\frac{\partial AH^2}{\partial x_i} \approx 2H.\frac{\partial AH}{\partial x_i}$ (the water level changes are insignificant over time and space) and the correlation coefficients (α_i, β_i, δ_i, γ_i, θ_i) ≈ 1; then, the system of equations (24), (25), (26) arrive at the classic 2DH flow system of equations.

Conclusion

The 2DH flow system of equations derived from the dual approach, the system of equations (24), (25), (26), are more general than those received from the classical approach with the appearance of the general terms: $\frac{\partial AH^2}{\partial t_i}$; $\frac{\partial AH^2}{\partial x_i}$ on the right-hand side of the system of equations (24), (25), (26). As a result, it could be used to describe the flow characteristic better compared to the classic 2DH flow system of equations; especially when in the flow region appear unusual phenomena such as solid objects, liquid containing other added ingredients, external forces f_x, f_y, f_z, reversals...

The correlation coefficients (α_1, β_1, α_2, α_3, α_4, α_5, β_1, β_2, β_3, β_4, β_5) increase the adjustable ability of calculated results.

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