Research Article

Markov Chain Models for Occurrence of Monsoon Rainfall in Different Zones of Assam and Meghalaya

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Abstract

During the South West Monsoon season, Markov chain models of different orders are considered to understand the distribution patterns of Assam and Meghalaya, India. The study conducted with the relevant data of 140 years (1871 - 2010) of twelve meteorological stations spread over the states. Application of Akaike's Information Criteria revealed that first order Markov chain is the best for rainfall forecasting. A period of 2 - 5 days of occurrence of rainfall is observed for various stations. The mean recurrence time for both dry and wet days and also steady state probabilities are computed for different orders of Markov chain models. The observed and theoretical values of steady state probabilities are realistically matched.

Keywords: Markov Chain Model; Rainfall Probability; Stationary Probability; Akaike's Information Criteria; Mean Recurrence Time; Rainfall Forecasting

Introduction

Due to complex nature of natural phenomenon, one cannot explain precisely the natural hydrological phenomena. Rainfall is one of the prime sources of natural phenomenon that modify the development of crop, flood, drought and spread of diseases.

A proper management of water distribution, yield of crop, alertness of drought/flood and environmental issues are largely dependent on the probability of occurrence of rainfall. It is known that rainfall events are separated by interval of time with reasonably high correlation. A time series of either occurrence or non-occurrence of rainfall on a day at a location is examined by several investigators, namely, Tyagi., *et al*. [1], Iyenger and Basak [2], Chatterjee., *et al*. [3], Khambete and Biswas [4] and others by different methods of statistical, numerical and synoptic techniques. In the recent past, time series analysis was utilized by Sengupta and Basak [5], Iyenger [6], Basak [7,8]. The utility of Markov chain model application is to ascertain forecast of weather states (dry or wet) of the near future with help of current state.

Markov chain model in determining weather condition has been attempted by several researchers. Mimikou [9] (opined that second order auto-regressive model is suitable for monthly sums of wet days than sum of daily precipitation obtained from Markov chain. SØrup., *et al*. [10] reported theoretically that first order Markov chain is very important, instead the 2nd order Markov chain is found to be significant; it is followed by models developed models for Sri Lanka [11]. Hossain and Anam [12] upon analysing rainfall data of Bangladesh concluded that wet days of previous two time periods indulges the wet days of current time during season.

Basak [13] utilized Markov chain model for a comprehensive analysis in West Bengal and pointed out that first order chain is suitable for the state. Tetty., *et al*. [14] utilized Markov chain to identify locations in south eastern coast of Ghana. In Haryana, India, 3-state Markov chain model with 5 independent parameters was utilized by Aneja and Srivastava [15] and also Jones and Thronton [16] with reasonably good result. In Orissa, India, the first order Markov chain model can be suitably represent the precipitation pattern as reported by Dash [17]. Yoo., *et al*. [18] utilized Markov chain decomposition to identify climatic change impact. Jimoh and Webster [19] noticed that Akaike information criteria (AIC) estimates are consistently more effective than Bayesian Information Criteria (BIC) for an order of Markov chain and no discernible difference persists between the model parameters of 1st and 2nd order. Other efforts on modelling on Markov chain performed in the past are, namely, Dasgupta and De [20], Thiagarajan., *et al*. [21], Ghose

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Dastidar., *et al.* [22], Pant and Shivhare [23] and Senthivelan., *et al* [24]. The Markov chain models have few advantages: firstly, forecasts are based on prediction on local information. Also, the minimal computation of processing of data is necessary for prediction. For short length of record, lower order chain representation yields suitable fit than higher order chains. However, the weather state in Assam and Meghalaya region is, so far, not adequately studied. In the paper, sequence of rainfall, namely, dry and wet days over 12 stations of Assam and Meghalaya studied with Markov chain models for prediction.

The yield of crop pattern particularly after rainfall condition mostly depends on rainfall characteristics. A criterion related the phenomenon, like wet and dry spells could be used for analyzing rainfall to obtain specific vital information needed for crop planning and carrying out agricultural operation [9,25]. Certain relationship exists between present day state and preceding day state; that is developed and utilized by Markov chain models. The numbers of preceding days under consideration is the order of the Markov chain. The order of the Markov chain is the count of preceding days taken into account. Many researchers [25,26] utilized the model the daily occurrence of rainfall. These studies support that first order Markov chain optimally explain the weather state. In this paper, the referred direction is followed for the weather state of the region of Assam and Meghalaya in the monsoonal seasonal.

Data and Method of analysis

The daily rainfall at twelve meteorological stations in different zones of Assam and Meghalaya, namely, Dhubri (26.02°N, 88.97°E), Golaghata (26.52°N, 93.66°E), Sibsagar (26.98°N, 94.64°E), Nowgong (25.06°N, 79.44°E), North Lakhimpur (27.22°N, 94.10°E), Nalbari (26.36°N, 91.33°E), Silchar (24.83°N, 92.77°E), Halflong (25.16°N, 93.01°E), Barpeta (26.32°N, 90.98°E), Goalpara (26.08°N, 86.37°E), Dibrugarh (27.47°N, 86.37°E) and Gauhati (26.14°N, 91.74°E) are considered.

Daily rainfall data of South West Monsoonal season (June-September) for the period of 140 years (1871 - 2010) is analysed in the paper. The data is categorized as binary representing DD and WD (dry days and wet days respectively) in terms random variable. The missing data, if any, are distributed randomly in terms of DD and WD. The binary data representing DD and WD days are well represented in terms of discrete random variable as:

X_k = 0, if rainfall does not occur on the kth day

= 1, if rainfall occurs on the kth day

where, k = 1, 2, ..., etc.

Each of the would be categorized as DD or WD (0 or 1) for proposed two-state Markov chain For the following data, it remains in the same state or proceed towards other states; that is transition takes place. The probability of such a transition is extracted as follows:

Let {Yt, t ∈ T} to be a two-state Markov chain with index set T and state space S [0, 1] represented as DD and WD respectively. Then, transition probability for two-state first order Markov chain is [27]:

$P_{ij} = \{X_{t+1} = j \mid X_t = i\}$

The transition probabilities for two-state second order and third order Markov chains in the above notation is

$P_{ijk} = \{X_{t+1} = k \mid X_t = j, X_{t-1} = i\}$

$P_{ijkl} = \{X_{t+1} = l \mid X_t = k, X_{t-1} = j, X_{t-2} = i\}$

The two-state Markov chain of any order is completely determined by its initial state and a set of transition probabilities P_{ij}, P_{ijk}, P_{ijkl}, which are estimated using conditional relative frequencies.

For each of the year (sample), the three orders of Markov chain, the parameter estimates are computed and the overall estimates are average of the estimate of the sample. Akaike Information Criteria (AIC) is utilized in assessing the order of the two state Markov chain. As per the criteria, for a given s-state, Markov chain of order 'm' is the most appropriate model, if it minimizes the AIC function:

$AIC_m = -2L_m + 2sm(s - 1)$

where

$$L_0 = \sum_{i=0}^{s-1} n_j \ln(P_j)$$

$$L_1 = \sum_{i=0}^{s-1} \sum_{j=0}^{s-1} n_{ij} \ln(P_{ij})$$

$$L_2 = \sum_{i=0}^{s-1} \sum_{j=0}^{s-1} \sum_{k=0}^{s-1} n_{ijk} \ln(P_{ijk})$$

$$L_3 = \sum_{i=0}^{s-1} \sum_{j=0}^{s-1} \sum_{k=0}^{s-1} \sum_{l=0}^{s-1} n_{ijkl} \ln(P_{ijkl})$$

At the selected twelve stations, the observed and expected number of dry and wet spells of different orders is compared for the chosen data period 1871 - 2000, using the Chi-square test [28]. The whole period of data (1871 - 2010) is fragmented into two parts. The first part (1871 - 2000) of data is utilized for modelling part and last 10 years data (2001 - 2010) is kept isolated for validation of the model

It is relevant to consider n-step probabilities obtained by using first order Markov chain. For Markov chain of first order, probability matrix P containing one-step transition probabilities are

computed. The transition probabilities of the chain ate extracted with the elements of the matrix Pⁿ [22,25]. The transition probabilities of the elements of the matrix P^n becomes constant as $n \to \infty$. Usually, it is observed that 5 - 6 steps, the transition probabilities and are independent of the initial state.

These steady-state probabilities are noted as:

π_0 = Steady state probabilities of DD

π_1 = Steady state probabilities of WD

Utilizing the conditional probability on the Markov chain of first order using computational formula:

$\pi_1 = P01/(1 + P01 + P11)$

$h\pi_0 = 1 - \pi_1$

The Markov chain of second order may be computed as:

$\pi_0 = (P10.P100 + P11.P110)/(1 - P00.P000 + P10.P100 - P01.P010 + P11.P110)$

$\pi_1 = 1 - \pi_0$

The expressions for the chain of higher orders are though computed but not presented due to its cumbersome nature.

Results and Discussion

The statistical analysis has been done for each year separately. However, results discussed are then to average of 130 years (1871 - 2000) under consideration. The estimated transition probability P00, meaning that DD is followed by DD is estimated. In a similar way, other estimates P01, P10 and P11 are computed for all the 12 stations and are presented in table 1a. It is realized from the table that for the first order Markov chain considering all the stations, the probability of WD followed by WD (P11) is observed to be highest (varies from 0.6492 to 0.8833), then the probability of DD followed by DD (P00) that varies from 0.4477 to 0.6736; it is followed by the probabilities of WD followed by DD (P10) and DD followed by WD (P01) where magnitude lies between the magnitudes between the corresponding values of P00 and P11.

Station	P00	P01	P10	P11
Dhubri	0.5898	0.4102	0.2645	0.7355
Golaghata	0.4477	0.5522	0.2588	0.7412
Sibsagar	0.5560	0.4439	0.1692	0.8308
Nowgong	0.5469	0.4531	0.2654	0.7346
North Lakhimpur	0.5624	0.4376	0.2057	0.7943
Nalbari	0.6414	0.3585	0.3643	0.6357
Silchar	0.4871	0.5129	0.1316	0.8683
Halflong	0.4936	0.5063	0.1971	0.8029
Barpeta	0.6736	0.3263	0.3507	0.6492
Goalpara	0.6223	0.3776	0.2767	0.7233
Dibrugarh	0.5117	0.4882	0.1555	0.8444
Gauhati	0.5750	0.4249	0.3203	0.6797

Table 1a: Estimation of first order transition probabilities of two state Markov chains.

The transition probabilities of second order Markov chains are presented in table 1b. The transition probability of P001 lies in between 0.1443 to 0.6708 over the stations. The other transition probabilities which are reasonably high are P111 (varies from 0.2649 to 0.7647). Instead, the transition probabilities which are reasonably low are P000 (varies from 0.2132 to 0.5639) and P100 (varies from 0.0659 to 0.4814). Broadly, the probabilities indicate that in the monsoonal season, WDs are more frequent than DDs. The feature is reflected in both orders of Markov chain. The probability that rainy day after a rainy day after day is highest (such as P11, P111 and P001) compared to the rest of the transition probabilities.

Station	P000	P100	P010	P001	P111	P110	P101	P011
Dhubri	0.3705	0.1407	0.1399	0.5811	0.5606	0.1759	0.1229	0.2731
Golaghata	0.2132	0.1102	0.1746	0.4432	0.5647	0.1757	0.1494	0.3786
Sibsagar	0.3809	0.0659	0.1017	0.5517	0.7008	0.1305	0.1028	0.3426
Nowgong	0.3298	0.1278	0.1473	0.5402	0.5549	0.1791	0.1382	0.3083
North Lakhimpur	0.3330	0.1075	0.1119	0.5568	0.6413	0.1524	0.0987	0.3269
Nalbari	0.2559	0.0584	0.1068	0.4755	0.7647	0.1044	0.0724	0.4099
Silchar	0.2648	0.0875	0.1350	0.4853	0.6591	0.1444	0.1090	0.3753
Halflong	0.4708	0.2177	0.1334	0.6708	0.441	0.2084	0.1323	0.1936
Barpeta	0.4038	0.1589	0.1481	0.6178	0.5555	0.1689	0.1165	0.2315
Goalpara	0.2715	0.0767	0.0988	0.5037	0.7201	0.1235	0.0795	0.3898
Dibrugarh	0.3406	0.1749	0.1633	0.5639	0.4814	0.1993	0.1443	0.2649
Gauhati	0.5639	0.4814	0.1993	0.1443	0.2649	0.1402	0.1099	0.3215

Table 1b: Estimation of second order transition probabilities of two state Markov chains.

The Akaike Information Criteria (AIC) values for the first and second order Markov chain with incorporating transition probabilities, namely, transition counts and are presented in table 2.

The table indicates that two-state first order chain minimizes the AIC criteria.

Station	Order I		Order II	
	LI	**AIC1**	**LII**	**AICII**
Dhubri	-5145.2363	10294.4727	-10113.6455	20235.2911
Golaghata	-4788.7959	9581.5918	-9299.4785	18606.9572
Sibsagar	-4788.7959	9581.5918	8518.0381	-17044.0762
Nowgong	-4872.2690	9748.5381	-9632.3066	19272.6133
North Lakhimpur	-4034.0273	8072.0547	-7959.2910	15926.5823
Nalbari	-4987.0063	9978.0127	-9708.7451	19425.4902
Silchar	-3817.8989	7639.7978	-7549.9941	15107.9883
Halflong	-9244.083	18496.1661	-4667.4853	9338.9707
Barpeta	-4719.9883	9443.9766	-9129.5215	18267.0431
Goalpara	-4204.4209	8412.8418	-8185.7939	16379.5879
Dibrugarh	-3297.9287	6599.8574	-6538.2222	13084.4443
Gauhati	-4721.9902	9447.9805	-6538.2222	13084.4443

Table 2: AIC scores for the model of different orders at twelve stations.

As per the Markov chain of first order, the transition probabilities of expected number of spells for DD and WD have been computed for the period 2001-2010. The corresponding observed values are also calculated for the period. Chi-square test has been applied to ascertain the goodness of fit of the two sets of data (Table 3). The test reveals that the test is accepted in 216 out of 228 cases.

	2001		2002		2003		2004		2005	
	Dry	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**
Dhubri	6.2512	9.3211	3.4411	4.7691	5.1222	8.0502	3.4312	1.3716	2.4104	13.5902
Golaghata	5.2611	3.9118	9.5794	3.4704	5.8705	9.5894	3.6968	9.5394	3.6967	15.805
Sibsagar	6.1243	0.3 544	5.0263	6.8331	7.4075	5.5536	4.8350	0.6749	-	-
Nowgong	8.2014	0.0678	9.9478	3.8044	3.6966	1.1087	5.5596	2.6028	5.3923	7.0130
North Lakhimpur	5.8705	9.7894	3.4500	4.2819	5.2222	9.0409	3.3142	1.8314	2.0410	18.5095
Nalbari	5.2611	3.9118	9.5894	3.4704	3.3964	5.8705	1.7097	5.5512	6.3967	6.1301
Silchar	6.1243	12.3544	5.0263	6.8331	6.1877	2.6028	5.5536	7.4075	14.8350	.6749
Halflong	9.7941	5.8705	3.4567	4.2819	5.2222	9.0309	1.8344	6.3147	2.6310	8.6095
Barpeta	2.6028	6.1877	5.0263	6.8331	-	-	7.4075	5.5536	4.8350	0.6749
Goalpara	9.6522	15.217	6.1236	7.2198	3.0987	1.3456	2.3158	3.1932	5.1299	8.1244
Dibrugarh	9.1243	6.1243	10.3544	5.0263	6.8331	6.1877	2.6028	5.5536	7.4075	4.83500
Gauhati	6.1243	13.3 44	15.0263	6.8331	7.4075	5.5536	4.8350	0.6749	-	-
	2006		2007		2008		2009		2010	
	Dry	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**
Dhubri	6.2512	9.3211	3.4411	4.7691	5.1222	8.0502	3.4312	1.3716	2.4104	13.5902
Golaghata	5.2611	3.9118	9.5794	3.4704	5.8705	9.5894	3.6968	9.5394	3.6967	5.8051
Sibsagar	6.1243	0.3 544	5.0263	6.8331	7.4075	5.5536	4.8350	0.6749	-	-
Nowgong	8.2014	0.0678	12.9478	3.8044	3.6966	1.1087	5.5596	2.6028	5.3923	7.0130
North Lakhimpur	5.8705	9.7894	3.4500	4.2819	5.2222	9.0409	3.3142	1.8314	2.0410	18.5095
Nalbari	5.2611	3.9118	9.5894	3.4704	3.3964	5.8705	1.7097	5.5512	6.3967	6.1301
Silchar	6.1243	12.3544	5.0263	6.8331	6.1877	2.6028	5.5536	7.4075	4.83500	.6749
Halflong	11.7941	5.8705	3.4567	4.2819	5.2222	9.0309	1.8344	6.3147	2.6310	18.6095
Barpeta	2.6028	6.1877	5.0263	6.8331	-	-	7.4075	5.5536	4.8350	0.6749
Goalpara	9.6522	15.2179	6.1236	7.2198	3.0987	1.3456	2.3158	3.1932	5.1299	8.1244
Dibrugarh	6.1243	0.3544	5.0263	6.8331	7.4075	5.5536	4.8350	0.6749	-	-
Gauhati	6.1243	11.3544	5.0263	6.8331	6.1877	2.6028	5.5536	7.4075	4.83500	.6749

Table 3: Goodness of fit test for observed and expected count (for first order Markov chain) of dry and wet spells for test data.

*Upper 5% value of Chi-square distribution with 5 degrees of freedom is 11.07.

The n-0step transition probability matrix P^n stabilizes after 5 - 6 iterations (P standing as one-step transition matrix as discussed) are very much realistic and simulates the first order chains of forecasting DD and WD perhaps better than any order [17,25]. The steady state probabilities as obtained from the n-step probabilities. The results are presented in table 4. It is seen that these are same in most of the cases.

The mean recurrence time which is the reciprocal of steady state probabilities from computational formula of DD and WD are presented in tables 5a and 5b. As an example, in Dhubri, mean recurrence time for DD and WD are 1.9303 and 1.6448. A comparison is done of such mean recurrence time for the observed data period 2001 - 2010 for both DDs and WDs from formula and observation period in the respective cases. Those corresponding values are found to be closely matched.

Conclusions

For south west monsoon rainfall data for 130 years (1871 - 2000) in twelve regional stations of Assam and Meghalaya are analysed as a two-state Markov chain model of both first and second order in order to understand DD and WD probabilistic pattern. The analysis points out the following results.

Station	Stationary probability of dry day		Stationary probability for wet day	
	From matrix	From first order	From matrix	From first order
Dhubri	0.3920	0.3920	0.6079	0.6079
Golaghata	0.3191	0.3191	0.6809	0.6809
Sibsagar	0.2759	0.2759	0.7241	0.7241
Nowgong,	0.3694	0.3694	0.6306	0.6309
North Laldumpur	0.3197	0.3197	0.6802	0.6802
Nalbari	0.5039	0.5039	0.4960	0.4906
Silchar	0.2043	0.2043	0.7957	0.7957
Halflong	0.2802	0.2802	0.7198	0.7198
Barpeta	0.5180	0.5180	0.4819	0.7198
Goalpara	0.4229	0.4229	0.5771	0.5771
Dibrugarh	0.2416	0.2416	0.7583	0.5771
Gauhati	0.4298	0.4298	0.5702	0.5702

Table 4: Comparison of stationary probabilities obtained from N-step transition matrix and computational formula on a first order chain

Station	From Markov Model		Observed mean recurrence										
	Stationary Probability	Mean recurrence time	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	
Dhubri	0.3920	1.9303	2.13	1.99	1.91	1.97	2.10	2.01	2.05	2.17	1.87	1.96	2.01
Golaghata	0.3191	2.3648	2.41	2.36	2.11	2.44	2.48	2.38	2.41	2.18	2.42	2.69	2.37
Sibsagar	0.2759	2.3648	2.41	2.42	2.96	2.34	2.31	2.31	2.41	2.36	2.64	2.14	2.49
Nowgong	0.3694	4.1348	3.08	4.03	3.96	3.84	3.71	3.98	4.29	4.17	4.21	3.97	3.91
North Lakhimpur	0.3197	2.3268	2.31	2.51	2.44	2.51	2.84	2.31	2.66	2.44	2.67	2.61	2.39
Nalbari	0.5039	1.9842	2.21	2.01	2.11	2.13	2.41	1.74	1.84	1.99	1.98	1.74	2.04
Silchar	0.2043	4.8931	4.10	4.22	4.88	3.97	4.99	5.01	4.77	4.21	4.11	4.51	4.97
Halflong	0.2802	3.5692	3.55	3.87	3.88	3.74	4.01	3.54	3.52	3.62	3.66	3.77	3.97
Barpeta	0.5180	1.9303	1.99	1.87	1.77	1.66	1.87	2.11	2.77	2.90	2.92	1.97	1.88
Goalpara	0.4229	2.3647	2.39	2.35	2.14	2.41	2.51	2.36	2.47	2.36	2.47	2.45	2.31
Dibrugarh	0.2416	4.1348	4.12	4.18	3.98	3.99	4.05	3.88	3.74	4.12	4.22	4.13	4.11
Gauhati	0.4298	2.3267	2.33	2.11	2.05	2.45	2.88	2.77	2.41	2.55	2.14	2.97	2.11

Table 5a: Mean recurrence time for dry days from stationary probability at different stations.

Station	From Markov Model		Observed mean recurrence										
	Stationary Probability	Mean recurrence time	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	
Dhubri	0.6079	1.6448	1.55	1.54	1.98	1.87	1.66	2.71	1.55	1.64	1.81	1.57	1.66
Golaghata	0.6809	1.4686	1.51	1.64	1.87	1.44	1.55	1.64	1.49	1.47	1.07	1.47	1.12
Sibsagar	0.7240	1.3810	1.39	1.45	1.38	1.38	1.38	1.39	1.74	1.84	1.38	1.87	2.01
Nowgong	0.6306	1.5858	1.55	1.58	1.28	1.57	1.56	1.59	1.59	1.66	1.54	1.57	1.71
North Lakhimpur	0.6802	1.4700	1.44	1.48	1.59	1.41	1.47	1.58	1.44	1.55	1.47	1.67	1.47
Nalbari	0.4960	2.0160	1.88	1.97	1.98	2.31	2.31	2.06	2.05	2.74	1.97	1.98	2.03
Silchar	0.7957	1.2567	1.25	1.21	1.41	1.38	1.31	1.24	1.26	1.32	1.38	1.26	1.31
Halflong	0.7198	1.3892	1.41	1.24	1.37	1.41	1.38	1.39	1.41	1.32	1.39	1.31	1.41
Barpeta	0.4819	2.0748	2.17	2.07	2.01	2.51	2.22	2.41	2.08	2.09	2.01	2.14	2.15
Goalpara	0.5771	1.7327	1.77	1.98	1.77	1.66	1.51	1.55	1.74	1.52	1.61	1.73	1.21
Dibrugarh	0.7583	1.3189	1.41	1.55	1.54	1.31	1.41	1.39	1.44	1.61	1.33	1.51	1.22
Gauhati	0.5702	1.7536	1.77	1.88	1.25	1.88	1.66	1.91	2.01	1.74	2.11	2.01	1.81

Table 5b: Mean recurrence time for wet days from stationary probability at different stations.

Two-state first order Markov chain is most suitable explaining the DD and WD days. The realistic situation that weather spells (DD or WD) possess shorter length more frequently, but longer spells are of less frequent.

The transition probabilities P11, P00, P001 and P11 are much less than the remaining transition probabilities for both first and second order Markov chains; meaning that WD situation is highly frequent with an exception of state of DD, a two day persistence in dry days. The result is very relevant for monsoonal season.

The probability of rainfall on a given day is in the range 0.4410 and 0.7533 for first order chain over the stations.

The concerned probabilities achieved from the second order chain are very near to that of first order chain. It may concluded that the steady state probabilities are independent of the initial state and order of the Markov chain, claiming an important finding of the Markov chain analysis.

For the first and second order Markov chain, probability of the occurrence of DD following WD, namely, P10, P100, P101 over the state of Assam and Meghalaya is observed to be very low which is in relevance to the nature of SWM season.

The first order Markov chain satisfactorily describe the process of analysis of DD and WD over time as endorsed by the sensibly close values of the observed and theoretical values of mean recurrence times for the test data.

For first order two state Markov chain, AIC selection criteria indicate least value and are utilized to build up the model the model.

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